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A Note on the Scaling Limit of a Complete Open Surface

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1. Introduction.

It is interesting to study the geometric meaning of total curvature of complete open surfaces. The influence of the total curvature of a Riemannian plane on the Lebesgue measure of rays were investigated first by M. Maeda [3], [4], K. Shiga [5] and later by K. Shiohama, T. Shioya and M. Tanaka [6], etc. The author proved in [2] that a pointed Hausdorff approximation map between connected, complete and noncompact Riemannian 2-manifolds with finite total curvature has a natural continuous extension to their ideal boundaries with the Tits metrics. In view of the above results it is natural to expect that the scaling limit of such an M will be a flat cone generated by the ideal boundary $M(\infty)$ of M equipped with the Tits metric d_∞.

Let M be a connected, complete and noncompact Riemannian 2-manifold with a finite total curvature. The Huber theorem implies that M is finitely connected. A compact set $C \subset M$ is by definition a core of M iff $M \setminus \mathrm{Int}(C)$ consists of k tubes $U_1, \cdots, U_k$ such that each U_i is homeomorphic to $S^1 \times [0, \infty)$ and such that each ∂U_i is a piecewise smooth simple closed curve. If $\kappa(\partial U_i)$ is the total geodesic curvature of ∂U_i, then the Gauss-Bonnet theorem implies $c(C) + \sum_{i=1}^{k} \kappa(\partial U_i) = 2\pi\chi(M)$. Moreover

$$s_i := \kappa(\partial U_i) - c(U_i)$$

is nonnegative and independent of the choice of tubes having the same end as U_i and

$$2\pi\chi(M) - c(M) = \sum_{i=1}^{k} s_i\,.$$

In [9] T. Shioya proved that M admits an ideal boundary $M(\infty)$ with the Tits metric d_∞ such that $(M(\infty), d_\infty)$ is the union of circles with lengths $s_1, \cdots, s_k$.

Let d be the distance function induced from the Riemannian metric of M. We denote by $(M_t; o)$ for an arbitrary fixed point $o \in M$ and for $t > 0$ the scaling by t of the

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