TOKYO J. MATH.
VOL. 18, NO. 1, 1995

Transformation $\tilde{G}$ of Analytic Functionals with Unbounded Carriers and Its Applications

Kunio YOSHINO

Sophia University

1. Introduction.

In [1] and [8], Avanissian and Supper considered transformation $\tilde{G}$ for analytic functionals. They applied transformation $\tilde{G}$ to study arithmetic entire functions of exponential type in Abel sense and derived Abel summation formula for entire functions of exponential type in C^n by using the sequence $\{D^nF(n)\}$. They also showed some relations between analytic functionals and classical special functions using transformation $\tilde{G}$. In this paper we will consider transformation $\tilde{G}$ for analytic functionals *with unbounded carrier*. As application, we derive some theorems for holomorphic (non-entire) functions of exponential type defined in direct product of half planes by using the sequence $\{D^{-n}F(-n)\}$.

2. Notations.

$U=\{\zeta=r\exp(i\theta)\in C^n: 0\leq r<(\pi-|\theta|)/|\sin(\theta)|\}$. $\Phi(\zeta)=\exp(-\zeta)/\zeta$. Φ is biholomorphic mapping between $U/\{0\}$ and $C/[-e, 0]$ ([4]). We put $\psi=\Phi^{-1}$.

For set L in C^n, $L_i=\mathrm{pr}_i(L)$ denotes i-th projection of L. $\langle z, t\rangle=z_1t_1+\cdots+z_nt_n$ for $z=(z_1, \cdots, z_n)$, $t=(t_1, \cdots, t_n)\in C^n$. $H_L(z)=\sup_{t\in L}\mathrm{Re}\langle z, t\rangle$.

$d(F)$ denotes transfinite diameter of compact set F in C. For the details of transfinite diameter, we refer the reader to [6].

$$D_x^{-n}f(x)=\frac{1}{(n-1)!}\int_0^\infty f(x-y)y^{n-1}dy\,. \qquad (n\in N)$$

Under suitable conditions, $D_x^n D_x^{-n}f(x)=f(x)$ valids for ordinary differential operator D_x. We put

$$D^{-m}=D_{x_1}^{-m_1}\cdots D_{x_n}^{-m_n}\,,\quad |m|=m_1+\cdots+m_n \qquad \text{for } m=(m_1, \cdots, m_n)\in N^n\,.$$

$\mathrm{Exp}(D:L)=\{$holomorphic functions in D satisfying condition $(*)$ in theorem 2$\}$.

Received November 24, 1993