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On the Cohomology of Coxeter Groups and Their Finite Parabolic Subgroups

Toshiyuki AKITA

Osaka University

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1. Introduction.

The purpose of this paper is to study the cohomology of Coxeter groups in terms of their parabolic subgroups of finite order. Given finite sets S and $\{m_{ij}\}$, $(i, j) \in S \times S$, where m_{ij} are integers or ∞ such that $m_{ii}=1$, $2 \leq m_{ij}=m_{ji} \leq \infty$ $(i \neq j)$, the group W defined by the generators $\{r_i\}_{i \in S}$ and the fundamental relation $(r_i r_j)^{m_{ij}}=1$, $m_{ij} \neq \infty$, is called a *Coxeter group*. We will identify the set of generators $\{r_i\}_{i \in S}$ with the set S. Also, if we wish to emphasise the set S we shall write (W, S) in place of W. (Some authors call (W, S) a *Coxeter system*.)

A subgroup of (W, S) generated by a subset $T \subseteq S$ is called a *parabolic subgroup* of W and is denoted by W_T. In particular, $W_S = W$ and $W_\varnothing$ is the trivial subgroup. A parabolic subgroup inherits a structure of a Coxeter group in an obvious way. Note that Coxeter groups of finite order are completely classified. The reader will refer to [1] or [6] for a general theory of Coxeter groups.

Given a Coxeter group (W, S), let $\mathscr{F}$ be the poset of (possibly empty) subsets F of S such that W_F is a finite parabolic subgroup of W. Given a W-module A of coefficients, set

$$\mathscr{H}^*(W, A) = \operatorname*{lim.inv.}_{F \in \mathscr{F}} H^*(W_F, A),$$

where the inverse limit is taken with respect to the restriction maps $H^*(W_F, A) \to H^*(W_{F'}, A)$ (where $F \supset F'$), and define

$$\rho : H^*(W, A) \to \mathscr{H}^*(W, A) \tag{1}$$

to be the canonical homomorphism induced by the restriction maps $H^*(W, A) \to H^*(W_F, A)$. If $A = k$ is a commutative ring with unity (regarded as a W-module with the trivial W-action), then $\mathscr{H}^*(W, k)$ is a graded ring and ρ is a ring homomorphism.

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