TOKYO J. MATH.
VOL. 18, NO. 1, 1995

Arithmetic Holomorphic Functions of Exponential Type on the Product of Half Planes

Kunio YOSHINO

Sophia University

1. Introduction and notations.

In [2] Bazylewicz considered arithmetic entire functions of exponential type in C^m (i.e. $f(N^m) \subset \mathcal{O}_A$, $\mathcal{O}_A$ is the set of algebraic integers.). In this paper we generalize Bazylewicz's result to holomorphic functions of exponential type in the product of half planes using the theory of analytic functionals with non-compact carrier. We adhere the following notations. N is the set of integers greater than 1. Let K be a number field over Q with degree $[K: Q]=d=r+2s$. Let $K^{(i)}$ $(1 \leq i \leq r)$ be real conjugate fields and $\overline{K}^{(r+j)}=K^{(r+s+j)}$ $(1 \leq j \leq s)$ be complex conjugate fields of K. We put $\delta=d$ if $K \subset R$, $\delta=d/2$ if $K \not\subset R$. For an algebraic integer α, we put $\overline{|\alpha|}=\max|\alpha_i|$ (where α_i's are conjugates of α over Q). $\mathcal{O}_A$ denotes the ring of algebraic integers and $\mathcal{O}_K$ is the ring of algebraic integers in K. For $p(z)=\sum a_n z^n \in K[z]$, we put $p^{(j)}(z)=\sum a_n^{(j)} z^n \in K^{(j)}[z]$, where $a_n^{(j)}$ are conjugates of a_n. $\tau(F)$ denotes the transfinite diameter of compact set F in C. For the details of transfinite diameter, we refer the reader to [1]. $\overline{F}$ is complex conjugate of $F \subset C$. $C-F$ is the complement of F in C. S^a denotes closure of S. $H_L(z)=\sup_{\zeta \in L} \mathrm{Re}\langle z, \zeta\rangle$ is the supporting function of L in C^m, where $\langle z, \zeta\rangle=z_1\zeta_1+\cdots+z_m\zeta_m$ for $z=(z_1, \cdots, z_m)$, $\zeta=(\zeta_1, \cdots, \zeta_m) \in C^m$. $L_i=\mathrm{pr}_i(L)$ is i-th projection of set L in C^m.

Following Theorem 1 is our main result.

THEOREM 1. *There exists a finite set* ϑ of $\mathcal{O}_A^m$ *(direct product of* $\mathcal{O}_A$*) having following property: Suppose that* $0 \leq k' < 1$ *and* $f(z)$ *satisfies*

(i) $f(z)$ *is holomorphic in* $\prod_{i=1}^m \{z_i: \ \mathrm{Re} z_i < -k'\}$,

(ii) *For any* $\varepsilon > 0$ *and* $\varepsilon' > 0$, *there exists* $C_{\varepsilon,\varepsilon'} \geq 0$ *such that*

$$|f(z)| \leq C_{\varepsilon,\varepsilon'} \exp(H_L(z)+\varepsilon|z|) \qquad (\mathrm{Re} z_i \leq -k'-\varepsilon'),$$

where L is a closed convex set contained in

$$\prod_{i=1}^m \{\zeta_i \in C: |\mathrm{Im}\zeta_i| \leq b_i < \pi, \ \mathrm{Re}\zeta_i \geq a_i\},$$

Received October 26, 1993
Revised July 12, 1994