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Differential Forms on Ringed Spaces of Valuation Rings

Koji SEKIGUCHI

Sophia University

In memory of Professor Yukiyosi Kawada

Introduction.

Given a field K and a subring A of K, we consider the set of valuation rings of K which contain A. This set has a structure of a local ringed space, denoted by $\mathrm{Zar}(K|A)$ (see [6] or [7]).

In this paper, we shall show that normal integral schemes $(X, \mathcal{O}_X)$ proper over $\mathrm{Spec}\,A$ with rational function field K are quotient spaces of $\mathrm{Zar}(K|A)$ and $\mathcal{O}_X=\Phi_{X*}\mathcal{O}_Z$. Here $\mathcal{O}_Z$ is the structure sheaf of $Z=\mathrm{Zar}(K|A)$ and $\Phi_X: Z\to X$ is the quotient mapping. In order to show this, we introduce a category $\mathscr{C}_0(K|A)$ of local ringed spaces, which contains both $\mathrm{Zar}(K|A)$ and all integral schemes proper over $\mathrm{Spec}\,A$ with rational function field K (see Theorems 1 and 1′).

For objects X of $\mathscr{C}_0(K|A)$, we introduce sheaves Ω_X^m of differential forms as in the case of schemes over $\mathrm{Spec}\,A$. In particular if A is a perfect field and X is a regular scheme, then Ω_X^m coincides with the ordinary sheaf of regular differential forms and $\Omega_X^m=\Phi_{X*}\Omega_Z^m$ for any multi-index m (see Theorem 2). From this, the birational invariance of regular differential forms of regular varieties follows immediately.

To define structure sheaves on quotient spaces of $\mathrm{Zar}(K|A)$ and sheaves Ω_X^m on objects X of $\mathscr{C}_0(K|A)$ in a unified way, we shall introduce the notion of intersection sheaf in §0.

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§0. Let $\mathscr{A}$ be the category of A-modules or the category of A-rings, where A is a commutative ring with unity. For an object N of $\mathscr{A}$, we denote by $\mathrm{Sub}_{\mathscr{A}}(N)$ the totality of subobjects of N. For a subset E of N, we put

$$\mathrm{Sub}_{\mathscr{A}}(N|E)=\{M\in \mathrm{Sub}_{\mathscr{A}}(N) \mid E\subset M\}\ .$$

Let $(E_i)_{i\in I}$ be a family of subsets of N. Then

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