TOKYO J. MATH.
VOL. 18, NO. 1, 1995

Compact Space-Like m-Submanifolds in a Pseudo-Riemannian Sphere $S_p^{m+p}(c)$

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(Communicated by T. Nagano)

Dedicated to Professor Tsunero Takahashi on his 60th birthday

Introduction.

In this paper, we shall consider the problem whether or not there exists a compact space-like m-dimensional submanifold in a pseudo-Riemannian sphere $S_p^{m+p}(c)$ with parallel mean curvature vector which is not totally umbilic.

A pseudo-Riemannian sphere $S_p^{m+p}(c)$ is an $(m+p)$-dimensional indefinite Riemannian space of index p and with constant curvature $c>0$, which is constructed in a pseudo-Euclidean space R_p^{m+1+p} as follows. First, a pseudo-Euclidean space R_p^{m+p+1} is of real $(m+p+1)$-tuples $x=(x_1, \cdots, x_{m+p+1})$ with scalar product

$$\langle x, y\rangle=\sum_{i=1}^{m+1} x_i y_i-\sum_{\alpha=m+2}^{m+p+1} x_\alpha y_\alpha .$$

Then

$$S_p^{m+p}(c)=\{x\in R_p^{m+p+1} \mid \langle x, x\rangle=1/c\} .$$

In the special case $p=1$, we call $S_1^{m+1}(c)$ a de Sitter space.

Let us consider M a compact space-like m-dimensional submanifold in $S_p^{m+p}(c)$. Then M is diffeomorphic to a Riemannian sphere S^m. (See Lemma 1 in §1). Here, M is totally umbilic if and only if M is a space-like $(m+1)$-plane section in $S_p^{m+p}(c)$, and then, M is congruent to a Riemannian sphere $S^m(c')$ of constant curvature c' where $c\geq c'>0$.

Montiel [9] has proved that a compact space-like hypersurface M in a de Sitter space $S_1^{m+1}(c)$ is totally umbilic if the mean curvature H of M is constant.

So we have been considering the higher codimensional case, and gotten the following.

Received July 6, 1993
Revised October 18, 1993