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On the Rational Approximations to $\tanh\frac{1}{k}$

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Introduction.

Let k and N_1 be positive integers with $N_1 \geq 10$, and let p_n/q_n be the n-th convergent of $\tanh(1/k)$. Let γ_{N_1}, δ_n, and $\gamma^*_{N_1}$ be defined by

$$\gamma_{N_1}=2\left(k+\frac{k+1}{N_1-1/2}\right)\left(1+\frac{\log\log(2k(N_1+1)/e)}{\log(N_1+1)}\right),$$

$$\delta_n=\frac{(k(2n+1)+2)\log\log q_n}{\log q_n},$$

and

$$\gamma^*_{N_1}=\max\{\delta_n \mid 1\leq n<N_1\},$$

respectively.

In the previous paper [1], we proved the following.

THEOREM A. *Let $k\geq 2$ be positive integers. Then*

$$\left|\tanh\frac{1}{k}-\frac{p}{q}\right|>\frac{\log\log q}{\gamma q^2\log q}$$

for all integers p and q with $q\geq 2$, where

$$\gamma\geq\max\{\gamma_{N_1}, \gamma^*_{N_1}\}$$

for any positive integer $N_1\geq 10$.

COROLLARY A. *For all integers p and q with $q\geq 2$,*

$$\left|\tanh\frac{1}{2}-\frac{p}{q}\right|>\frac{\log\log q}{6q^2\log q}.$$

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