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On Certain Multiple Series with Functional Equation in a Totally Imaginary Number Field I

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§1. Introduction.

In the recent paper [3], we considered a multiple series in a totally real number field, which is regarded as a generalization of the double series

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{m}e^{-2\pi nm\tau}\qquad(\mathrm{Re}\,\tau>0)\,,$$

and proved that it satisfies functional equation.

In the present paper, we shall treat analogous problem in a totally imaginary number field. Our method will be similar to that of [3]; the proof is based on the transformation formula of Hecke-Rademacher, the expression of our series by integrals and the calculation of residues.

Let K be a totally imaginary number field of degree $n=2r$, $K^{(p)}$, $K^{(r+p)}=\bar{K}^{(p)}$ $(p=1,\cdots,r)$ the conjugates of K. Let $\mathfrak{d}$ be the differente ideal of K, $D=N(\mathfrak{d})$ the absolute value of the discriminant of K and R the regulator of K.

If μ is a number of K, then we denote by $\mu^{(q)}$ the conjugates of μ in $K^{(q)}$ $(q=1,\cdots,n)$. We define n-dimensional vector $\mu=(\mu^{(1)},\cdots,\mu^{(n)})$. More generally, we shall often use n-dimensional complex vector $\xi=(\xi_1,\cdots,\xi_n)$ such that $\xi_{r+p}=\bar{\xi}_p$ $(p=1,\cdots,r)$ and write

$$S(\xi)=\sum_{q=1}^{n}\xi_q\,,\qquad N(\xi)=\prod_{q=1}^{n}\xi_q\,.$$

Let $\tau_1,\cdots,\tau_n$ be positive numbers such that $\tau_{r+p}=\tau_p$ $(p=1,\cdots,r)$. Let $\xi=(\xi_1,\cdots,\xi_n)$ be a complex vector stated above. Let $\mathfrak{a}$ and $\mathfrak{b}$ be non-zero fractional ideals of K. For these τ, ξ, $\mathfrak{a}$ and $\mathfrak{b}$, we define the series $M(\tau,\xi\,;\mathfrak{a},\mathfrak{b})$ as follows:

$$M(\tau,\xi\,;\mathfrak{a},\mathfrak{b})=\sum_{\substack{(\mu)\subset\mathfrak{a}\\(\mu)\neq 0}}\frac{1}{N(\mu)^{1/2}}\sum_{\substack{\nu\in\mathfrak{b}\\ \nu\neq 0}}\exp\{-2\pi S(|\nu\mu|\tau)+2\pi iS(\mu\nu\xi)\}\,,\tag{1.1}$$

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