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On Certain Multiple Series with Functional Equation in a Totally Real Number Field I

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§1. Introduction.

In the analytic theory of partition function, the double series

$$f(\tau)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m}e^{-2\pi mn\tau}\qquad(\mathrm{Re}\,\tau>0) \tag{1.1}$$

plays an important role. It is well-known that $f(\tau)$ satisfies the functional equation:

$$f(\tau)-\frac{\pi}{12\tau}-\frac{1}{4}\log\tau=f\left(\frac{1}{\tau}\right)-\frac{\pi}{12}\tau-\frac{1}{4}\log\frac{1}{\tau}. \tag{1.2}$$

This remarkable equation has been proved by various methods (cf. Chandrasekharan [1, p. 170] or Schoenfeld [5]).

In this paper, we shall consider a multiple series that is a generalization of (1.1) in a totally real number field and prove that it satisfies a functional equation.

Let K be a totally real number field of degree n, $K^{(q)}$ $(q=1,\cdots,n)$ the conjugates of K. Let $\mathfrak{d}$ be the differente ideal of K, $D=N(\mathfrak{d})$ (norm of $\mathfrak{d}$) the absolute value of the discriminant of K, and R the regulator of K.

If μ is a number of K, then we denote by $\mu^{(q)}$ the conjugates of μ in $K^{(q)}$ $(q=1,\cdots,n)$. We define n-dimensional vector $\mu=(\mu^{(1)},\cdots,\mu^{(n)})$. More generally, we shall often use n-dimensional complex vector $\xi=(\xi_1,\cdots,\xi_n)$. For such ξ we put

$$S(\xi)=\sum_{q=1}^{n}\xi_q\,,\qquad N(\xi)=\prod_{q=1}^{n}\xi_q\,.$$

Let $\tau_1,\cdots,\tau_n$ be complex numbers with positive real parts. Let $\mathfrak{a}$ and $\mathfrak{b}$ be the fractional ideals of K. For such $\mathfrak{a}$, $\mathfrak{b}$ and $\tau_1,\cdots,\tau_n$, we define the series $M(\tau;\mathfrak{a},\mathfrak{b})$ as follows:

$$M(\tau;\mathfrak{a},\mathfrak{b})=\sum_{\substack{(\mu)\subset\mathfrak{a}\\(\mu)\neq0}}\frac{1}{|N(\mu)|}\sum_{\substack{\nu\subset\mathfrak{b}\\\nu\neq0}}\exp\{-2\pi S(|\mu\nu|\tau)\}\,, \tag{1.3}$$

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