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Noetherian Rings Graded by an Abelian Group

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Dedicated to Professor Takeshi Ishikawa on his 60th birthday

Introduction.

Throughout this paper, all rings are assumed to be commutative with identity.

Let G be an Abelian group. We say that a ring R is a G-graded ring, if there exists a family $\{R_g\}_{g\in G}$ of additive subgroups of R such that $R=\bigoplus_{g\in G}R_g$ and $R_gR_h\subset R_{g+h}$ for every $g, h\in G$. Similarly, a G-graded R-module is an R-module M for which there is given a family $\{M_g\}_{g\in G}$ of additive subgroups of M such that $M=\bigoplus_{g\in G}M_g$ and $R_gM_h\subset M_{g+h}$ for every $g, h\in G$.

The investigation of the ring-theoretic property of graded rings started with the following question of Nagata [13].

If G is the ring of integers $\boldsymbol{Z}$, then is Cohen-Macaulay property of R determined by their local data at graded prime ideals?

As is well-known, Matijevic-Roberts [12] and Hochster-Ratliff [8] gave an affirmative answer to the conjecture as above. Similarly Aoyama-Goto [1] and Matijevic [11] showed that the same as above is also true for Gorenstein property. Furthermore Goto-Watanabe developed a theory of $\boldsymbol{Z}^n$-graded rings and modules in their papers [5] and [6] and proved the relation between Bass numbers of graded modules at nongraded prime ideals and Bass numbers at graded prime ideals.

In this paper, we study G-graded rings and G-graded modules for an arbitrary Abelian group G.

Some homological properties of a G-graded ring R depend only on their local data at graded prime ideals, when $G=\boldsymbol{Z}^n$. But, for an arbitrary Abelian group G, informations about graded prime ideals are not enough to determine homological properties. For example, the hypersurface $k[X]/(X^2-1)$ is a $\boldsymbol{Z}_2$-graded ring by $\deg(X)=1\in\boldsymbol{Z}_2$ and has no graded prime ideals. Here $\boldsymbol{Z}_2=\boldsymbol{Z}/2\boldsymbol{Z}$. Therefore we introduce the notion of G-prime ideals as follows and improve Goto-Watanabe's arguments using this notion.

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