TOKYO J. MATH.
VOL. 18, NO. 1, 1995

On the Schur Indices of $SU_{l+1}(F_q)$ and $Spin_{2l}^-(F_q)$

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(Communicated by T. Nagano)

Dedicated to Professor Tosiro Tsuzuku

Introduction.

Let G be a connected, reductive algebraic group defined over a finite field F_q with q elements of characteristic p and let F be the corresponding Frobenius endomorphism of G. As usual, G^F denotes the finite group of F-fixed points of G (G^F = the group of F_q-rational points). Let B be an F-stable Borel subgroup of G and U be its unipotent radical. Then U is also F-stable and U^F is a Sylow p-subgroup of G^F.

In [4] Gel'fand and Graev found that when $G=SL_n$ (or GL_n) any irreducible character of G^F occurs with non-zero multiplicity in some induced characters λ^{G^F} where λ runs over all the linear characters of U^F and if λ is "in general position" then λ^{G^F} is multiplicity-free. The latter "multiplicity-one theorem" holds for a general G (Yokonuma [19], Steinberg [16]) but the former fact does not hold for a general G (e.g. Sp_4). However it seems that almost all the irreducible characters of G^F occur in $\sum_\lambda \lambda^{G^F}$.

R. Gow has initiated to investigate the rationality-properties of the characters λ^{G^F} in order to get informations about the Schur indices of the irreducible characters of G^F ([5, 6], also cf. [7]).

In the rest of this introduction we assume that p is not a bad prime for G for the sake of simplicity. In [13] we studied the rationality of the λ^{G^F} generally and saw that any λ^{G^F} takes values in $k=Q(\sqrt{(-1)^{(p-1)/2}p})$ (we assume that $p\neq 2$) and is realizable in k_v for any finite place v of k. From this it follows that if χ is an irreducible character of G^F such that $\langle\chi, \lambda^{G^F}\rangle_{G^F}=1$ for some λ or $p\nmid\chi(1)$ then the Schur index $m_Q(\chi)$ of χ with respect to Q is at most two. In [14] we announced some more detailed results when G is a simple algebraic group. Main purpose of this paper is to give their proofs when G is a twisted group. For the sake of simplicity we shall assume that G is simply-connected. As the cases $G={}^3D_4$, ${}^2E_6$ are treated in [13] we assume here that $G=SU_{l+1}$ or $Spin_{2l}^-$.

Received May 10, 1993
Revised November 7, 1994