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Symplectic Geometry and Deformation of Infinite Dimensional Cycles Associated to Cauchy-Riemann Operators

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1. Introduction.

Let M be a Riemann surface, and P be a principal bundle over M with structure group $U(n)$. For every connection A on P we can associate $\bar{\partial}_A$-operator as follows. We denote by $\Omega_C^1(M, \mathrm{ad}(P))$ the complex vector space of 1-forms over M valued on $\mathrm{ad}(P)$. In parallel with the decomposition of this vector space:

$$\Omega_C^1(M, \mathrm{ad}(P)) = \Omega^{1,0}(M, \mathrm{ad}(P)) \oplus \Omega^{0,1}(M, \mathrm{ad}(P)),$$

the covariant derivative d_A decomposes into ∂_A and $\bar{\partial}_A$:

$$d_A = \partial_A + \bar{\partial}_A,$$

$$\partial_A : \Omega_C^0(M, \mathrm{ad}(P)) \to \Omega^{1,0}(M, \mathrm{ad}(P)),$$

$$\bar{\partial}_A : \Omega_C^0(M, \mathrm{ad}(P)) \to \Omega^{0,1}(M, \mathrm{ad}(P)).$$

Hence the Cauchy-Riemann operator $\bar{\partial}_A$ is associated to each connection A. We have therefore a family of Fredholm operators $\{\bar{\partial}_A\}_{A \in \mathscr{A}}$ parametrize by $\mathscr{A}$:

$$\mathscr{A} \ni A \mapsto \bar{\partial}_A.$$

Let $\mathscr{G}$ denote the gauge transformation group. Due to the gauge invariance of $\{\bar{\partial}_A\}_{A \in \mathscr{A}}$, if we take as a parameter space the quotient space $\mathscr{A}/\mathscr{G}$ instead of $\mathscr{A}$, then we see that the above association can be reduced to $\mathscr{A}/\mathscr{G}$. Thus we get a new family of operators

$$\mathscr{A}/\mathscr{G} \ni [A] \mapsto \bar{\partial}_{[A]}.$$

This defines an element of k-theory over $\mathscr{A}/\mathscr{G}$ (see [2], [5], [10]).

These correspondences among operators $\{\bar{\partial}_A\}$, $\{\bar{\partial}_{[A]}\}$ and parameter spaces $\mathscr{A}$, $\mathscr{A}/\mathscr{G}$ are easily generalized to a general situation, i.e., a correspondence with Fredholm

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