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Compact Homomorphisms on Algebras of Continuous Functions

Junzo WADA

Waseda University

Introduction.

The purpose of this note is to study compact and weakly compact homomorphisms between algebras of continuous functions. For a completely regular Hausdorff space S, we denote by $C(S)$ the algebra of all complex-valued continuous functions on S endowed with its compact-open topology. M. Lindström and J. Llavana [4] gave characterizations of compact and weakly compact homomorphisms from $C(S)$ to $C(T)$, where T and S are completely regular Hausdorff spaces. Let A and B be closed subalgebras of $C(S)$ and $C(T)$ respectively. Here we study compact and weakly compact homomorphisms φ from A to B.

After some preliminaries in §1, we introduce in §2 closed subalgebras of some type which are called function algebras induced by uniform algebras. These subalgebras contain $C(S)$ and algebras of analytic functions. We discuss in §2 compactness and weak compactness of φ in the case A is a function algebra induced by a uniform algebra and φ is a composition operator. We give conditions under φ is compact or weakly compact and establish the relationship between compactness and weak compactness of φ.

§1. Preliminaries.

For a completely regular Hausdorff space X, we denote by $C(X)$ the algebra of all complex-valued continuous functions on X endowed with its compact-open topology. Throughout this note we let S and T denote completely regular Hausdorff spaces.

Let A and B be subalgebras of $C(S)$ and $C(T)$ respectively. Then we easily have the following (cf. [6], [8]).

(a) Let φ be a continuous linear operator from A to B. Then there is a continuous mapping τ from T to the dual space A' of A with respect to the w^*-topology $\sigma(A', A)$ such that

$$(*) \qquad [\varphi(f)](y)=\tau(y)(f)\,, \qquad f\in A \text{ and } y\in T\,.$$

(b) Let φ be a continuous homomorphism from A to B. Then there is a continuous

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