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Kähler Magnetic Flows for a Manifold of Constant Holomorphic Sectional Curvature

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Introduction.

In his paper [7], being inspired by a classical treatment of static magnetic fields in the three dimensional Euclidean space, T. Sunada studied the flow associated with a magnetic field on a Riemann surface. A closed 2-form $\boldsymbol{B}$ on a complete Riemannian manifold M is called a *magnetic field*. Let $\Omega=\Omega_{\boldsymbol{B}}$ denote the skew symmetric operator on the tangent bundle TM of M satisfying $\boldsymbol{B}(u, v)=\langle u, \Omega(v)\rangle$ with the Riemannian metric $\langle\ ,\ \rangle$ for every tangent vectors u and v. The Newton equation on this setting is of the form $\nabla_{\dot{\gamma}}\dot{\gamma}=\Omega(\dot{\gamma})$ for a smooth curve γ on M. We call such a curve satisfying this equation a *trajectory* for $\boldsymbol{B}$. In terms of physics it is a trajectory of a charged particle moving on this manifold under the action of the magnetic field. The aim of this paper is to give a light in terms of magnetic fields on dynamical systems for a manifold of complex space form. The most important dynamical object associated to a Riemannian manifold is the geodesic flow. Consider the case without an action of magnetic field, $\boldsymbol{B}=0$. The Newton equation turns out to $\nabla_{\dot{\gamma}}\dot{\gamma}=0$, hence trajectories are nothing but geodesics. In the same way as the geodesic flow corresponds to geodesics, we can define a flow associated with a magnetic field in the following manner. One can easily check that every trajectory $\gamma(t)$ for $\boldsymbol{B}$ has constant speed, hence is defined for $-\infty<t<\infty$. We call a trajectory *normal* if it is parametrized by its arc length. The *magnetic flow* $\boldsymbol{B}\varphi_t$: $UM\to UM$ on the unit tangent bundle UM is defined by

$$\boldsymbol{B}\varphi_t(v)=\gamma_v(t)\,,\qquad v\in UM\,,\quad -\infty<t<\infty\,,$$

where γ_v denotes the normal trajectory for $\boldsymbol{B}$ with $\dot{\gamma}_v(0)=v$. If γ is a trajectory for $\boldsymbol{B}$, then the curve $\sigma(t)=\gamma(\lambda t)$ with a constant λ is a trajectory for $\lambda\boldsymbol{B}$. This represents a dynamical property of trajectories for $\boldsymbol{B}$.

On a Riemann surface, magnetic fields are of the form $f\cdot\mathrm{Vol}$ with a smooth

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