TOKYO J. MATH.
VOL. 18, NO. 2, 1995

Lower Bounds for the Class Number and the Caliber of Certain Real Quadratic Fields

Hedi AMARA

The University of Tunis

(Communicated by T. Nagano)

Introduction.

We give some canonical cycles of reduced ideals for the real quadratic fields $K=\boldsymbol{Q}(\sqrt{m})$ with $m=4q^2+1$, $m=q^2+4$ (q odd), $m=q^2+1$ (q odd) and $m=q^2\pm 2$ (q odd). Lower bounds of the class number $h(K)$ and the caliber Cal(K) (number of reduced ideals) are given. Some of those lower bounds for the class number are obtained, by other methods, in [2].

Let m be a square free integer, K the quadratic field $\boldsymbol{Q}(\sqrt{m})$, $\theta=(1+\sqrt{m})/2$ if $m\equiv 1$ (4) and $\sqrt{m}$ otherwise, θ^τ the conjugate of θ, $F(X)=(X-\theta)(X-\theta^\tau)$ the fundamental polynom of K and $D(K)=(\theta-\theta^\tau)^2$ the discriminant of K. Every reduced ideal $\mathfrak{a}$ of K is presented in the form $[a, \theta-c]$ where a is the norm of $\mathfrak{a}$, c an integer such that $0<\theta-c<a$ and $F(c)=-ab$. $\mathfrak{a}=[a, \theta-c]$ is reduced when $(a+b)^2\leq D(K)$ and a cycle of reduced ideals starts from $\mathfrak{a}$ to the reduced ideal $\mathfrak{a}_1=((\theta^\tau-c)/a)\mathfrak{a}=[b, \theta-c_1]$, this operation is repeated until we obtain $\mathfrak{a}$ another time (see [1]). The class number of K is equal to the number of cycles and the caliber is the sum of the numbers counting reduced ideals in every cycle. In what follows $\tau(x)$ denotes the number of distinct positive divisors of the integer x.

I. $\boldsymbol{m=4q^2+1}$, $F(X)=X^2-X-q^2$. Let d be a proper divisor of q, and put $q=\lambda d$. From

$$F(1)=-q^2, \qquad F(q)=-q=-d\lambda,$$

$$F(q+1-\lambda)=-\lambda(2q+1-d-\lambda), \qquad F(q+1-d)=-d(2q+1-d-\lambda),$$

we construct the cycles:

$$\text{-}[1, \theta-q]\rightarrow[q, \theta-1]\rightarrow[q, \theta-q],$$

$$\text{-}[d, \theta-q]\rightarrow[\lambda, \theta-(q+1-\lambda)]\rightarrow[2q+1-\lambda-d, \theta-(q+1-d)].$$

Received June 3, 1994