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Ringed Spaces of Valuation Rings over Hilbert Rings

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Introduction.

Given a field K and a subring A of K, we consider the local ringed space $\mathrm{Zar}(K|A)$ consisting of all valuation rings of K which contain A (see [4] or [5]). If A is a Hilbert ring, in other words, if any prime ideals of A are intersections of maximal ideals (see [1], p. 373), then the ringed space $X=\mathrm{Zar}(K|A)$ satisfies the condition

$$(1) \qquad \beta_X: t(X_{\mathrm{cl}}, \mathscr{F}_X|_{X_{\mathrm{cl}}}) \xrightarrow{\sim} (X, \mathscr{F}_X)\,.$$

Here X_{cl} is the set of closed points of X and $\mathscr{F}_X$ is the structure sheaf on X. For the morphism β_X of ringed spaces, see (17). Given a topological space W, we denote by tW the set of irreducible closed subsets of W. If $(W, \mathscr{F}_W)$ is a ringed space, then tW also has a structure of ringed spaces donoted by $t(W, \mathscr{F}_W)$. The correspondence $(W, \mathscr{F}_W)\mapsto t(W, \mathscr{F}_W)$ gives rise to a covariant functor from the category of ringed spaces to itself. Moreover, if W is a T_1-space, then the ringed space $(X, \mathscr{F}_X)=t(W, \mathscr{F}_W)$ satisfies the condition (1), and the morphism $f: X\to Y$ of ringed spaces obtained by t from a morphism of T_1-ringed spaces satisfies the condition

$$(2) \qquad f(X_{\mathrm{cl}})\subset Y_{\mathrm{cl}}\,.$$

In this case, t gives an equivalence of the categories (see section 1). Therefore, we shall consider the following problem.

PROBLEM 1. Characterize the ringed spaces $(X, \mathscr{F}_X)$ satisfying the condition (1).

EXAMPLES. (i) Let X be an affine scheme $\mathrm{Spec}\,A$. Then X satisfies the condition (1) if and only if A is a Hilbert ring.

(ii) Any integral scheme X of finite type over a field satisfies the condition (1).

For a local ringed space $(W, \mathscr{O}_W)$, we introduce a morphism $\pi_W: W\to \mathrm{Spec}\,\mathscr{O}_W(W)$ defined by $\pi_W(x)=\rho_{W,x}^{-1}(m(\mathscr{O}_{W,x}))$. Here $\rho_{W,x}: \mathscr{O}_W(W)\to\mathscr{O}_{W,x}$ are the canonical mappings and $m(R)$ denotes the unique maximal ideal of a local ring R. The next problem is closely related to Problem 1.

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