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Zeta Function and Perron-Frobenius Operator of Piecewise Linear Transformations on R^k

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§1. Introduction.

In [7], we considered a piecewise linear transformation F on R^k into itself. Roughly speaking, we determined the eigenvalues of the Perron-Frobenius operator P corresponding to the transformation F by the zeros of the Fredholm determinant $\det(I-\Phi_n(z))$ (the definition of the Fredholm matrix $\Phi_n(z)$ will be given in the next section).

We denote by J^o, J^{cl} and $\Delta J=J^{cl}\setminus J^o$ be the inner, the closure and the boundary of a set J. Set

$$\xi=\liminf_{n\to\infty} \operatorname*{ess\,inf}_{x\in I} \frac{1}{n}\log|\det D(F^n)(x)|\,,$$

$$\lambda=\limsup_{n\to\infty} \sup_J \frac{1}{n}\log \#\{w\in\mathscr{W} : |w|=n, \langle w\rangle\cap\Delta J\neq\varnothing\}\,,$$

where $D(F^n)$ is the jacobian matrix of F^n, $\sup_J$ is the supremum over all possible convex polyhedrons J, $\mathscr{W}$ is a set of words, and $\langle w\rangle$ is a parallelopiped which corresponds to a word $w\in\mathscr{W}$ (see §2 for precise definitions). We say that F is expanding if

$$\liminf_{n\to\infty}\frac{1}{n} \operatorname*{ess\,inf}_{x\in I} \log\min|\text{the eigenvalue of } D(F^n)(x)|>0\,.$$

Our theorem in [7] is:

THEOREM A. *Assume that F is expanding and $\xi>\lambda$.*

(i) *Then for any $\varepsilon>0$, there exists an integer n_0 and for $n\geqslant n_0$ and $|z|<e^{\xi-\lambda-\varepsilon}$, z^{-1} belongs to the spectrum of the Perron-Frobenius operator P restricted to $\mathscr{B}$ if and only if*

$$\det(I-\Phi_n(z))=0\,.$$

(ii) *The eigenfunctions of P on L^1 associated with eigenvalues modulus 1 belong to $\mathscr{B}$.*

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