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Harmonic Analysis on Homogeneous Vector Bundles on Hyperbolic Spaces

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Introduction.

Harmonic analysis on hyperbolic spaces $X=U(p,q;\boldsymbol{F})/(U(1;\boldsymbol{F})\times U(p-1,q;\boldsymbol{F}))$ $(\boldsymbol{F}=\boldsymbol{R},\boldsymbol{C},\boldsymbol{H})$ has been studied by many people. Faraut [1], Limic, Niederle, and Raczka [12], Molčhanov [14], Rossmann [15], and Strichartz [23] proved the Plancherel formula on hyperbolic spaces. One method of proof is to use the explicit expression of K-finite eigenfunctions of the Laplacian. Schlichtkrull [18], Sekiguchi [19], and Shitikov [22] studied the Poisson transformation for hyperbolic spaces. Schlichtkrull and Shitikov used the explicit expressions of K-finite eigenfunctions of the Laplacian, and of K-finite functions in degenerate principal series representations.

In this paper we generalize these results for a homogeneous vector bundle on X associated with an irreducible representation δ of $U(1;\boldsymbol{F})$. The basic tools are K-finite functions.

The first result (Theorem 5.2) is the Plancherel formula on the associated vector bundle. We decompose every f in a dense subspace of $L^2(X,\delta)$, the space of L^2-sections of the associated vector bundle, in terms of eigenfunctions of the Laplacian. We describe the Plancherel measure explicitly in terms of the $\boldsymbol{c}$-function ((4.10), (4.11)) for the associated degenerate series representation.

The second result of this paper is the determination of the closed $U(p,q;\boldsymbol{F})$-invariant subspaces of the eigenspaces of the Laplacian on the associated vector bundle (Theorem 6.2), and of the image and the kernel of the Poisson transformation (Theorem 6.4). The result is also new in the Riemannian case, $p=1$.

The degenerate principal series representations, which correspond to the boundaries of hyperbolic spaces, have been studied by Molčhanov [13], Klimyk and Gruber [4, 5, 6, 7], Vilenkin and Klimyk [25], and Howe and Tan [3], by using the explicit K-decompositions. The Poisson transformation gives an intertwining operator from a degenerate principal series representation to an eigenspace of the Laplacian. We use their results to construct K-finite eigenfunctions of the Laplacian and invariant subspaces

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