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Nonexistence of Normal Quintic Abelian Surfaces in P^3

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Introduction.

Normal surfaces X_d of degree d in the complex projective 3-space P^3 have simple birational structure if d is small: X_1 and X_2 are rational, and X_3 is birationally equivalent to a ruled surface (for further details, see [B-W], [H-W]), since in general $K_{X_d} \simeq \mathcal{O}_{X_d}(d-4)$. Moreover X_4 is birationally equivalent to either a ruled or a K3 surface ([Um1]).

To the contrary, various X_d may occur if $d \geq 5$. If the singularity of X_d is mild, then X_d is birationally equivalent to a surface of general type, while X_d may be birationally equivalent to a ruled surface if it has severe singularity. Moreover there are examples of X_5 which are birationally equivalent to K3 surfaces, Enriques surfaces or general elliptic surfaces ([I], [Yan], [St], [K], [Um2], [Um3], [Um4]). This leads us to the question whether there exists an X_d which is birationally an abelian or a hyperelliptic surface or not. The purpose of this note is to answer this question in the case of $d=5$. We prove:

MAIN THEOREM. *No normal quintic surface in P^3 is birationally equivalent to an abelian or a hyperelliptic surface.*

Our proof of the theorem goes as follows. First we note that if a normal quintic surface $X=X_5$ is birationally an abelian or a hyperelliptic surface, then its minimal resolution $\tilde{X}$ is an at most 5-fold blowing-up $\mu: \tilde{X} \to \bar{X}$ of the non-singular minimal model $\bar{X}$. On the other hand, the pull-back of K_X to $\tilde{X}$ minus $K_{\tilde{X}}$ is an effective divisor $\tilde{D}$, which reflects the property of the singularity of X fairly well. Such property of $\tilde{D}$ and the condition of $\mu_* \tilde{D}$ as a divisor on an abelian or hyperelliptic surface finally lead us in every case to a contradiction.

CONJECTURE. No normal hypersurface in P^3 is birationally equivalent to an abelian surface.

Also for hyperelliptic surfaces we raise:

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