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A Note on Satake Parameters of Siegel Modular Forms of Degree 2

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Introduction.

For a positive integer k, let S_k be the space of all Siegel cusp forms of weight k on $Sp(2, \boldsymbol{Z})$. Suppose $f\in S_k$ is an eigenform, i.e., a non-zero common eigenfunction of the Hecke algebra. Then we define the spinor L-function attached to f by

(0.1) $$L(s,f,\underline{\mathrm{spin}}) := \prod_p \{(1-\alpha_{0,p}p^{-s})(1-\alpha_{0,p}\alpha_{1,p}p^{-s})(1-\alpha_{0,p}\alpha_{2,p}p^{-s})(1-\alpha_{0,p}\alpha_{1,p}\alpha_{2,p}p^{-s})\}^{-1}$$

and the standard L-function attached to f by

(0.2) $$L(s,f,\underline{\mathrm{st}}) := \prod_p \left\{(1-p^{-s})\prod_{j=1}^{2}(1-\alpha_{j,p}^{-1}p^{-s})(1-\alpha_{j,p}p^{-s})\right\}^{-1},$$

where p runs over all prime numbers and $\alpha_{j,p}$ $(0\le j\le 2)$ are the Satake p-parameters of f. The right-hand sides of (0.1) and (0.2) converge absolutely and locally uniformly for $\mathrm{Re}(s)$ sufficiently large.

For an indeterminate t, we put

$$H_p(t,f,\underline{\mathrm{spin}}) := (1-\alpha_{0,p}t)(1-\alpha_{0,p}\alpha_{1,p}t)(1-\alpha_{0,p}\alpha_{2,p}t)(1-\alpha_{0,p}\alpha_{1,p}\alpha_{2,p}t),$$

$$H_p(t,f,\underline{\mathrm{st}}) := (1-t)\prod_{j=1}^{2}(1-\alpha_{j,p}^{-1}t)(1-\alpha_{j,p}t),$$

where $H_p(t,f,\underline{\mathrm{spin}})$, $H_p(t,f,\underline{\mathrm{st}})\in \boldsymbol{R}[t]$.

DEFINITION. (cf. Kurokawa [9]) We say that $f\in S_k$ satisfies the Ramanujan–Petersson conjecture if the absolute values of the zeros of $H_p(t,f,\underline{\mathrm{spin}})$ are all equal to $p^{-(k-3/2)}$ for all p.

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