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Gevrey Classes on Compact Real Analytic Riemannian Manifolds

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Introduction.

On the Euclidean space Gevrey classes on open sets and closed sets with boundary are studied by many mathematicians. In the present paper, we define Gevrey classes on compact real analytic Riemannian manifolds (without boundary) and study their properties. Special attention will be paid for the cases of the sphere and the Lie sphere.

Let X be a compact real analytic Riemannian manifold with a Riemannian metric g. We denote by $\mathscr{E}(X)$ the space of infinitely differentiable functions on X equipped with the usual topology, by $\mathscr{A}(X)$ the space of analytic functions on X equipped with the usual inductive limit topology, and by $\mathscr{B}(X)=\mathscr{A}'(X)$ the space of hyperfunctions on X. We denote by $\|\cdot\|_{L^2}$ the L^2-norm with respect to the measure $d\mu$ corresponding to g. Let $s>0$, $h>0$ and let Δ_X be the Laplace-Beltrami operator corresponding to g on X. We define Gevrey classes $\mathscr{E}_{\{s\}}(X)$ and $\mathscr{E}_{(s)}(X)$ by

$$\mathscr{E}_{\{s\}}(X)=\operatorname*{ind\,lim}_{h\to\infty}\left\{f\in\mathscr{E}(X);\ \sup_k\frac{1}{(2k)!^s h^{2k}}\|\Delta_X^k f\|_{L_2}<\infty\right\},$$

$$\mathscr{E}_{(s)}(X)=\operatorname*{proj\,lim}_{h\to 0}\left\{f\in\mathscr{E}(X);\ \sup_k\frac{1}{(2k)!^s h^{2k}}\|\Delta_X^k f\|_{L_2}<\infty\right\},$$

respectively. $\mathscr{E}_{\{s\}}(X)$ is a DFS space and $\mathscr{E}_{(s)}(X)$ is an FS space. We denote their dual spaces by $\mathscr{E}'_{\{s\}}(X)$ and $\mathscr{E}'_{(s)}(X)$, respectively.

According to Roumieu [9], the definition of Gevrey classes on $\boldsymbol{R}^{n+1}$ with compact support by means of the Laplacian and the supremum norm is equivalent to the usual definition given in Komatsu [2].

In our definition, the L^2-norm may be replaced by the sup-norm. In fact, for $s=1$, Lions-Magenes [3] proved the equivalence and their argument is still valid for $s>0$. Moreover, [3] showed that $\mathscr{E}_{\{1\}}(X)$ is equal to $\mathscr{A}(X)$. Further, Hashizume-Minemura [1] characterized the spaces $\mathscr{A}(X)$ and $\mathscr{B}(X)$ by the growth behavior of the coefficients

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