TOKYO J. MATH.
VOL. 18, NO. 2, 1995

Values of the Unipotent Characters of the Chevalley Group of Type F_4 at Unipotent Elements

Reginaldo M. MARCELO and Ken-ichi SHINODA

Sophia University

§0. Introduction.

Let $\mathbf{G}$ be a connected reductive algebraic group with connected center and split over a finite field $\mathbf{F}_q$ of characteristic p, and F the corresponding Frobenius morphism. Then the subgroup $G=\mathbf{G}^F$ of $\mathbf{G}$ consisting of elements fixed by F is a finite Chevalley group. The table consisting of the values at unipotent elements of the unipotent characters of G which we shall simply call the unipotent character table of G, is an essential part of the character table of G.

If $\mathbf{G}$ has type F_4 and p is good for $\mathbf{G}$ (that is, $p\geq 5$), the problem of completing the unipotent character table of G is reduced to the determination of the values of one almost character; this was settled independently by N. Kawanaka [Ka2, Ka3] and G. Lusztig ([Lu3], cf. Remark 4.3 of this paper also). The purpose of this paper is to form the table in the cases $p=2$ or 3. And since the unipotent character table when $p\geq 5$ can be obtained at once from that when $p=3$ (cf. Remark 4.5), we could actually form this table as well.

Through the Fourier transform matrix introduced by G. Lusztig [Lu1], the determination of the values of irreducible characters of G is equivalent to the determination of the values of almost characters. And the almost characters are closely related to another set of class functions on G called the characteristic functions associated with character sheaves of G, also due to Lusztig [Lu2]. In fact, he conjectured that if p is almost good for $\mathbf{G}$ the characteristic functions coincide with the almost characters up to multiplication by a scalar. In [Sho4], T. Shoji proved Lusztig's conjecture in the case where $\mathbf{G}$ has connected center; moreover, he showed that for $\mathbf{G}$ of type F_4, Lusztig's conjecture also holds when $p=3$, and a weaker version of it holds when $p=2$ (cf. Theorem 1.7).

Now, $G=F_4(p^n)$ has 37 unipotent characters and using known results, we can compute the values at unipotent elements of 30 of the corresponding almost characters. Our problem therefore is to determine the values of the remaining 7 almost characters,

Received February 10, 1994