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A Sharp Symmetrization of the L^2-Well-Posed Mixed Problem for Regularly Hyperbolic Equations of Second Order

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Introduction.

We consider the mixed problem

$$
(\mathrm{P})\quad
\begin{cases}
L[u]=\dfrac{\partial^2 u}{\partial t^2}-2\displaystyle\sum_{j=1}^{n} h_j(t,x)\dfrac{\partial^2 u}{\partial t\partial x_j}-\sum_{i,j=1}^{n} a_{ij}(t,x)\dfrac{\partial^2 u}{\partial x_i\partial x_j} \\
\qquad +a_0(t,x)\dfrac{\partial u}{\partial t}+\displaystyle\sum_{j=1}^{n} a_j(t,x)\dfrac{\partial u}{\partial x_j}+e_0(t,x)u=f(t,x) \\
u(0,x)=u_0(x)\,, \qquad u_t(0,x)=u_1(x) \\
B[u]|_{x_1=0}=a_{11}(t,0,x')^{-1/2}\Big\{a_{11}(t,0,x')\dfrac{\partial u}{\partial x_1} \\
\qquad +\displaystyle\sum_{j=2}^{n} a_{1j}(t,0,x')\dfrac{\partial u}{\partial x_j}+h_1(t,0,x')\dfrac{\partial u}{\partial t}\Big\} \\
\qquad +\displaystyle\sum_{j=2}^{n} b_j(t,x')\dfrac{\partial u}{\partial x_j}-c(t,x')\left(1+\dfrac{h_1(t,0,x')^2}{a_{11}(t,0,x')}\right)^{1/2} \\
\qquad \cdot\Big\{\dfrac{\partial u}{\partial t}-\left(1+\dfrac{h_1(t,0,x')^2}{a_{11}(t,0,x')}\right)^{-1}\displaystyle\sum_{j=2}^{n}\Big(h_j(t,0,x') \\
\qquad -\dfrac{h_1(t,0,x')}{a_{11}(t,0,x')}a_{1j}(t,0,x')\Big)\dfrac{\partial u}{\partial x_j}\Big\} \\
\qquad +\gamma(t,x')u|_{x_1=0}=g(t,x') \\
(t,x)=(t,x_1,x')\in \boldsymbol{R}^1_+\times\boldsymbol{R}^1_+\times\boldsymbol{R}^{n-1}
\end{cases}
$$

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