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# Geometric Approach to Rigidity of Horocycles

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## 1. Introduction.

Horocycles are defined as submanifolds associated with the geodesic flow on the unit tangent bundle of a two-dimensional compact connected Riemannian manifold with negative curvature. The geodesic flow and the horocycles themselves are interesting subjects of investigations in the theory of dynamical systems. Our interest in this paper is to characterize a manifold with negative curvature via horocycles and we call this characterization rigidity of horocycles.

Let $N$ be a two-dimensional compact connected orientable Riemannian manifold with variable negative curvature, and let $M$ be its unit tangent bundle. The expanding horocycles on $M$ are obtained associated with the geodesic flow $g$. Let $N'$ and $M'$ be defined similarly. The result is the following:

THEOREM A. *If a homeomorphism $\varphi : M \to M'$ maps every expanding horocycle on $M$ onto an expanding horocycle on $M'$ and preserves their orientations, then $N$ and $N'$ are homothetic.*

In this theorem to be homothetic means that there exists a diffeomorphism from $N$ to $N'$ and the difference between the metric on $N'$ and the image of the metric on $N$ by this diffeomorphism is a constant multiple. The orientation of expanding horocycles is defined in §2.

From a point of view of characterization of a manifold with negative curvature via horocycle flows, *i.e.*, rigidity of horocycle flows, there exists a typical result by M. Ratner [R]. Let $N_c$, $N'_c$ be complete 2-dimensional Riemannian manifolds with constant negative curvature, and let $M_c$, $M'_c$ be their unit tangent bundles, respectively. If there is a measurable isomorphism between horocycle flows on $M_c$ and $M'_c$, then $N_c$ and $N'_c$ are isometric. This result provided the motivation of this work.

In this paper we give the proof of the following theorem.

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