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On the Sum of Four Cubes and a Product of k Factors

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1. Introduction.

Page [10] [11] and Hooley [6] investigated the asymptotic behaviour of the number of representations of a natural number as a sum of squares and products of two positive factors, and established an asymptotic formula for the number of representations in each case where it can exist.

We consider here similar problems for cubes instead of squares. As is mentioned in Hooley [7, p. 180], an asymptotic formula for each case with at least five cubes and a product, or with at least two products and a cube is obtained by standard application of the circle method of Hardy and Littlewood. Indeed, let $k\geq 2$ be an integer and let $\nu(N)$ denote the number of representations of N as the sum of t products of k positive factors and s cubes. The number $\nu(N)$ is investigated in Waring's problem when $t=0$, and in the additive divisor problem when $s=0$. Here we consider the case $t, s\geq 1$. We introduce the functions

$$F(\alpha)=\sum_{m\leq N^{1/3}} e(m^3\alpha)\,, \qquad D_k(\alpha)=\sum_{n\leq N} d_k(n)e(n\alpha)\,, \tag{1.1}$$

where $e(\alpha)=\exp(2\pi i\alpha)$ and $d_k(n)$ denotes the number of ways of expressing n as a product of k positive factors. We have

$$\nu(N)=\int_0^1 D_k(\alpha)^t F(\alpha)^s e(-N\alpha)d\alpha\,.$$

We divide the unit interval [0, 1] into "major" and "minor" arcs. We don't give here definitions of major and minor arcs exactly, but we only indicate that major arcs, say $\mathfrak{M}$, is a union of narrow neighbourhoods of rational numbers with smaller denominators, and that $\mathfrak{M}$ is defined so that the integral

$$\int_{\mathfrak{M}} D_k(\alpha)^t F(\alpha)^s e(-N\alpha)d\alpha$$

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