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Uniqueness for an Inverse Problem for the Wave Equation in the Half Space

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1. Introduction.

Let us consider the initial boundary value problem:

$$u_{tt}-\Delta_x u+b(x)u_t+a(x)u=0 \qquad \text{in } \mathbf{R}^n_+\times(0, T)\,, \tag{1.1}$$

$$u=u_t=0 \qquad \text{on } \mathbf{R}^n_+\{0\}\,, \tag{1.2}$$

$$\partial_z u(y, 0, t)=f(y, t) \qquad \text{on } \mathbf{R}^{n-1}\times(0, T)\,. \tag{1.3}$$

Here $\mathbf{R}^n_+$ $(n>1)$ is the half space:

$$\mathbf{R}^n_+=\{x=(y, z)\in\mathbf{R}^n : y\in\mathbf{R}^{n-1}, z\in\mathbf{R}, z>0\}\,,$$

and $a\in L^\infty(\mathbf{R}^n_+)$, $b\in C^2(\overline{\mathbf{R}^n_+})$. Throughout this article we assume that

$$a \text{ and } b \text{ are constants on } \{x : |x|>r\}\,. \tag{1.4}$$

We are interested in uniqueness results of a, b from the Neumann to Dirichlet map:

$$\Lambda(a, b):\ f\mapsto u(y, 0, t) \qquad \text{on } \mathbf{R}^{n-1}\times(0, T)\,.$$

In the case that $b\equiv 0$, Rakesh [6] showed that $\Lambda(a, 0)$ uniquely determines a if T is large enough. Some authors studied the problem stated above in the bounded domain case instead of the half space ([2], [3]). Rakesh pointed out that the proof for the bounded domain case does not apply to the half space one, but he succeeded to prove the uniqueness mentioned above by using the results of x-ray transform obtained by Hamaker, Smith, Solmon, and Wagner ([1]). In this article we show that the result of Rakesh can be extended for the mixed problem (1.1)–(1.3) by the methods which are used in [3]. By u_j and $\Lambda_j\equiv\Lambda(a_j, b_j)$, we denote solutions and the Neumann to Dirichlet maps to the problem (1.1)–(1.3) corresponding to $a=a_j$, $b=b_j$ $(j=1\,,2)$ respectively.

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