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# On the Banach Algebra $M(p, q)$ $(1 \leq p < q \leq \infty)$

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## 1. Introduction.

Let $G$ be an infinite compact abelian group, $\Gamma$ (or $\hat{G}$) the dual group, $M(G)$ the convolution measure algebra on $G$, and $L^p(G)$ the $L^p$-space with respect to the Haar measure $m_G$ on $G$ for $1 \leq p \leq \infty$. Also for $1 \leq p \leq q \leq \infty$ let $M(p, q)$ be the set of all translation invariant bounded linear operators from $L^p(G)$ to $L^q(G)$. For $\mu \in M(G)$, $\mu$ is called an $L^p$-improving measure, if $\mu \in M(r, s)$ for some $1 \leq r < s \leq \infty$ (cf. [5]). When $p < q$, $M(p, q)$ is a commutative Banach algebra without unit with the operator norm, and $M(p, p)$ is a commutative Banach algebra with unit.

The purpose of this paper is an investigation of Fourier multiplier algebra $M(p, q)$ $(1 \leq p < q \leq \infty)$.

Hatori [10] characterized $\Lambda(p)$-sets on $\hat{G}$ by using the Banach algebra $M(p, q)$ $(1 < p < q < \infty)$ (cf. [3], [4]). Also, he characterized the maximal ideal space of $M(p, q)$ $(1 < p < q < \infty)$. These results are showed by applying Stone-Čech's compactification.

In §2, we give proofs of Theorems 2.2 and 2.5 that are simple proofs of his results, by the method of [12] and [20] without using Stone-Čech's compactification.

Igari-Sato [12] studied the operating function of $M(p, q)$ $(1 \leq p \leq q \leq \infty)$. The domain of the operating function is $[-1, 1]$. In §3, we investigate the operating function whose domain is the complex plane. When $G$ is the unit circle, our result is an extension of Rider's result [14] (cf. [16]).

There are many papers [5], [15], etc. about $L^p$-improving measures. But it seems that it is unknown about $L^p$-improving measures on thin sets (cf. [5; Open questions]). In §4, we construct non $L^p$-improving measures on some independent set, that are in $M_0(G)$ that is the set of all bounded regular Borel measures whose Fourier-Stieltjes transforms vanish at infinity.

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