TOKYO J. MATH.
VOL. 20, NO. 1, 1997

Volumes of Compact Symmetric Spaces

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(Communicated by T. Nagano)

H. Freudenthal [4] defined the natural volume of a semi-simple compact Lie group G induced from the Killing form and gave a formula of the natural volume of G. S. A. Broughton [3] calculated the volume in the case of G a classical Lie group. Another volume formula of the semi-simple compact Lie groups has been studied by H. Urakawa [7] and I. G. Macdonald [6] in different ways, respectively.

If G/K is a compact symmetric space, then the Killing form of G also induces a natural volume of G/K. The volumes of the projective spaces are obtained by using Jacobi fields (cf. [2], [5]). In the previous paper [1] we calculated the volumes of the Hermitian exceptional symmetric spaces $EIII$, $EVII$ and the twister space $Z(EIX)$ of the exceptional symmetric space EIX by using the computations of the 1st Chern classes. From those results we can calculate the natural volumes of the compact symmetric spaces as follows:

symbol	space	volume
A_n	$SU(n+1)$	$\dfrac{2^{n(2n+5)/2}(n+1)^{(n+1)^2/2}}{1!\,2!\cdots n!}\pi^{n(n+3)/2}$
B_n	$Spin(2n+1)$	$\dfrac{2^{n(4n+5)/2+1}(2n-1)^{n(2n+1)/2}}{1!\,3!\cdots(2n-1)!}\pi^{n(n+1)}$
C_n	$Sp(n)$	$\dfrac{2^{n(3n+1)}(n+1)^{n(2n+1)/2}}{1!\,3!\cdots(2n-1)!}\pi^{n(n+1)}$
D_n	$Spin(2n)$	$\dfrac{2^{3n^2}(n-1)^{n(2n-1)/2}}{2!\,4!\cdots(2n-2)!}\pi^{n^2}$
G_2	G_2	$\dfrac{2^{26}3^2\sqrt{3}}{5}\pi^8$
F_4	F_4	$\dfrac{2^{52}\,3^{45}}{5^4\,7^2\,11}\pi^{28}$

Received June 23, 1995