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Asymptotics of Scattering Phases for the Schrödinger Operator with Magnetic Fields

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1. Introduction.

In this paper we shall consider the asymptotics of scattering phases of Schrödinger equation with magnetic fields. The equation is described as follows:

$$\sum_{j=1}^{d}\{i\partial_j+b_j(x)\}^2u+q(x)u=\lambda u\ .$$

In particular we do *not* assume that the scalar potential $q(x)$ and the vector potential $b_j(x)$ are spherically symmetric. D. R. Yafaev defined in [8] the scattering phases of Schrödinger equation

$$-\Delta u+q(x)u=\lambda u\ ,$$

which has the scalar potential $q(x)$ *without* spherical symmetry, and he studied the asymptotics. One of his results is that the asymptotics of scattering phases depend on the asymptotics of *the even part* $q_e(x)$ *of the scalar potential* (i.e., $q_e(x)=(q(x)+q(-x))/2$). So we shall extend the result to the case for Schödinger equation with magnetic fields.

Yafaev gave a definition of scattering phases related with the eigenvalue of the modified scattering matrix $\Sigma(\lambda)$. $\Sigma(\lambda)$ is defined by $S(\lambda)J$ where $S(\lambda)$ is a scattering matrix and J is a reflection operator. In fact, the scattering phases defined by D. R. Yafaev make sense in physical point of view (cf. [9]). In a similar way we can also define scattering phases of Schrödinger equation with magnetic fields. In our case we can find that the asymptotics of scattering phases depend on the asymptotics of *the even part* $q_e(x)$ *of scalar potential* and *the odd part* $b_{j,o}$ *of the vector potential* (i.e., $b_{j,o}(x)=(b_j(x)-b_j(-x))/2$).

This paper is organized as follows. In section 2 we give the correct definition of the scattering phases related to the eigenvalues of the operator $\Sigma(\lambda)$. And we give main theorems without proofs. In section 3 some properties of compact operators are given. In particular the properties of singular values play an important role in this paper. In

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