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Formulations of Elliptic Lie Algebra $\hat{sl}(2)$ and Elliptic Virasoro Algebra by Vertex Operators

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1. Introduction.

In the definition of a vertex operator algebra ([5]), the Virasoro algebra comes out naturally and the Virasoro algebra is realized by Sugawara construction in terms of the affine Lie algebra, so towards a generalization of a vertex operator algebra in the case of elliptic Lie algebras, in this paper we give a formulation of elliptic Lie algebra $\hat{sl}(2)$ and the elliptic Virasoro algebra by vertex operators. Elliptic Lie algebra is a Lie algebra associated with an elliptic root system ([1]), which also called extended affine Lie algebra ([2], [3]), or 2-toroidal Lie algebra ([6]), and the elliptic Virasoro algebra, we call so for the reason of the correspondence with Lie algebra, the 2-dimensional Virasoro-Bott algebra ([8], [9]). A certain formulation and representation of 2-toroidal Lie algebras by vertex operators are already given by [3], [6], [7]. However, we give another formulation of elliptic Lie algebra $\hat{sl}(2)$ and the elliptic Virasoro algebra by vertex operators from the view point of a generalization of vertex operator formalism. At first we recall the definition and formulation of affine Lie algebras and the Virasoro algebra by vertex operators ([5]), and next with similar correspondence, we describe them in elliptic case.

2. Affine Lie algebras and Virasoro algebra.

We recall the definition of the affine Lie algebras and Virasoro algebra and their formulations by vertex operators [5]. We describe explicitly in the case of $sl(2, \mathbf{C})$ for simplicity, however the result is applicable for any finite dimensional simple Lie algebras. We set $\mathfrak{a}=sl(2, \mathbf{C})$, and choose the basis α_1, x_{α_1}, $x_{-\alpha_1}$ of $\mathfrak{a}$, where

$$\alpha_1=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \qquad x_{\alpha_1}=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \qquad x_{-\alpha_1}=\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}.$$

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