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On Certain Multiple Series with Functional Equation in a Totally Real Number Field II

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1. Introduction.

This is a continuation of the preceding paper [3] with the same title. We shall treat the series with "fractional powers" (see (1.1) below), which is a generalization of the series

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{n}e^{-2\pi m^k n\tau} \qquad (\tau>0)\,.$$

Let K be a totally real number field of degree n and $K^{(q)}$ $(q=1, \cdots, n)$ be the conjugates of K. Let μ be a number of K. We denote by $\mu^{(q)}$ the conjugates of μ in $K^{(q)}$ $(q=1, \cdots, n)$ and define n-dimensional vector $\mu=(\mu^{(1)}, \cdots, \mu^{(n)})$ correspondingly. More generally, we consider n-dimensional complex vectors $\xi=(\xi_1, \cdots, \xi_n)$. For such ξ we put

$$S(\xi)=\sum_{q=1}^{n}\xi_q\,, \qquad N(\xi)=\prod_{q=1}^{n}\xi_q\,.$$

Let U be the unit group of K and $U^k=\{\varepsilon^k \mid \varepsilon\in U\}$ the subgroup of U consisting of the k-th powers of units. Let α, β be the numbers of K. If α/β is an element of U^k, we say that α and β are associated with respect to U^k. Let k and l be coprime positive integers and $\tau_1, \cdots, \tau_n$ be non-zero complex numbers such that

$$|\arg\tau_q|<\frac{\pi}{2k} \qquad (q=1, \cdots, n)\,.$$

Let $\mathfrak{a}$ and $\mathfrak{b}$ be non-zero fractional ideals of K.

For such numbers $\tau_1, \cdots, \tau_n, k, l$ and ideals $\mathfrak{a}$, $\mathfrak{b}$, we shall define the series as follows:

$$(1.1) \qquad M(\tau; \mathfrak{a}, \mathfrak{b}; k, l)=M(\tau_1, \cdots, \tau_n; \mathfrak{a}, \mathfrak{b}; k, l)$$
$$=\sum_{\substack{\mu/U^k\\ 0\neq\mu\in\mathfrak{a}}}\frac{1}{|N(\mu)|}\sum_{0\neq\nu\in\mathfrak{b}}\exp\{-2\pi S(|\nu|^{k/l}|\mu|\tau^k)\}\,,$$

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