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The Integral Representations of Harmonic Polynomials in the Case of $\mathfrak{su}(p, 1)$

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Introduction.

Let $\mathfrak{g}$ be a complex reductive Lie algebra and let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be the complexification of a Cartan decomposition of $\mathfrak{g}_{\mathbf{R}}$, where $\mathfrak{g}_{\mathbf{R}}$ is a noncompact real form of $\mathfrak{g}$. Kostant-Rallis [3] showed that polynomials on $\mathfrak{p}$ are expressed as the tensor product of harmonic polynomials and K-invariant polynomials, where $K=\exp \operatorname{ad}\mathfrak{k}$. Related to this result, we showed in [14] that the set of common zero points of all K-invariant polynomials on $\mathfrak{p}$ is a uniqueness set of holomorphic functions on $\mathfrak{p}$ (see Proposition 1.1).

On the other hand, for classical harmonic functions on $\mathbf{C}^p$ and functions on the sphere, there are many studies (see, [2], [4], [5], [6], [7], [10], [12], [15], etc.). For example, it is known that harmonic functions on $\mathbf{C}^p$ are represented by an integral on some $O(p)$-orbits, and the reproducing kernels of these formulas are expressed by the Legendre polynomials (cf. Lemma 1.2). For details, see [7] Lemma 7 and [15] Theorem 2.4. In the Lie algebraic viewpoint, classical harmonic functions on $\mathbf{C}^p$ correspond to harmonic functions on $\mathfrak{p}$ for the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{so}(p, 1)$, and we can easily rewrite the classical integral formulas in Lemma 1.2 in the Lie algebraic form (A.1)–(A.4) in Appendix.

Our purpose of this paper is to obtain integral representation formulas of harmonic polynomials in the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{su}(p, 1)$. Our main results in this paper are described in Theorem 2.2, in which we obtain the similar results to the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{so}(p, 1)$. In the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{so}(p, 1)$ harmonic functions are expressed in the form of integral on some simple $K_{\mathbf{R}}$-orbits, where $K_{\mathbf{R}}=\exp \operatorname{ad}\mathfrak{k}_{\mathbf{R}}$. But in the case $\mathfrak{g}_{\mathbf{R}}=\mathfrak{su}(p, 1)$ we express the formulas in the form of double integrals on some family of $K_{\mathbf{R}}$-orbits.

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1. Preliminaries.

In this section we fix the notations and review known results. For details, see [2],

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