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On an n-th Order Linear Ordinary Differential Equation with a Turning-Singular Point

Minoru NAKANO

Keio University

Dedicated to Professor Toshihiko Nishimoto on his 60th birthday

1. Introduction.

1.1. We consider an n-th order linear ordinary differential equation

$$\varepsilon^{nh}y^{(n)}=\sum_{k=1}^{n}\varepsilon^{(n-k)h}p_k(x,\varepsilon)y^{(n-k)}\quad\left(0<|x|\le x_0,\ 0<\varepsilon\le\varepsilon_0,\ '=\frac{d}{dx}\right),\tag{1.1}$$

where x is a complex variable, and h, x_0 and ε_0 are positive constants.

The coefficients $p_k(x,\varepsilon)$'s are given by

$$p_k(x,\varepsilon):=p_k\cdot(x^m-\varepsilon^l/x^r)^k\qquad(k=1,2,\cdots,n)\,,\tag{1.1}'$$

where m, l and r are positive integers satisfying *the singular perturbation condition*:

$$h>\frac{m+1}{m+r}\,l\,,\tag{1.2}$$

and the constants p_k's are supposed to satisfy

$$\begin{cases} p_1:=\sum_{k=1}^{n}a_k\,,\quad p_2:=-\sum_{k_1<k_2}a_{k_1}a_{k_2}\,,\quad p_3:=\sum_{k_1<k_2<k_3}a_{k_1}a_{k_2}a_{k_3}\,,\\ \qquad\qquad\cdots\cdots\cdots\cdots \\ p_{n-1}:=(-1)^n\sum_{k_1<k_2<\cdots<k_{n-1}}a_{k_1}a_{k_2}\cdots a_{k_{n-1}}\,,\qquad p_n:=(-1)^{n+1}\prod_{k=1}^{n}a_k\,,\end{cases}\tag{1.3}$$

$$a_{k-1}<a_k\ (k=2,3,\cdots,n)\,;\qquad \forall a_k\neq0\,.\tag{1.4}$$

Accordingly, the characteristic equation of (1.1) is given by

$$L(x,\lambda)=0\,,\qquad L(x,\lambda):=\lambda^n-\sum_{k=1}^{n}p_k\cdot x^{km}\lambda^{n-k}=\prod_{k=1}^{n}(\lambda-a_kx^m)\tag{1.5}$$

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