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Timelike Surfaces of Constant Mean Curvature in Minkowski 3-Space

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Introduction.

Since the discovery of constant mean curvature tori in Euclidean 3-space, methods in the theory of integrable systems (or soliton theory) have been applied frequently in differential geometry. In particular, surfaces of constant mean or Gaussian curvature in Euclidean 3-space have been studied extensively [2], [3].

On the other hand, the geometry of surfaces in Minkowski 3-space has been a subject of wide interest [9], [10]. For example, a Kenmotsu-type representation formula for spacelike surfaces with prescribed mean curvature has been obtained by K. Akutagawa and S. Nishikawa, and M. A. Magid [9] has obtained such a representation formula for timelike surfaces. L. McNertney studied spacelike maximal surfaces, timelike extremal surfaces and timelike surfaces of constant positive Gaussian curvature by classical methods [10].

In our previous paper [5], we have studied spacelike surfaces with constant mean or Gaussian curvature in Minkowski 3-space via the theory of finite-type harmonic maps. To adapt the methods in the theory of integrable systems for our purposes, namely to study the geometry of surfaces, we reformulated the fundamental equations of spacelike surfaces. This allowed us to considerably simplify the computations in the study of such spacelike surfaces. The main tool of this reformulation was our use of split-quaternion numbers. In addition we obtained representation formulae for immersions in terms of loop group theory [5].

The purpose of this paper is to develop a corresponding setting for timelike surfaces of constant mean curvature in Minkowski 3-space.

In contrast to the case of spacelike surfaces or timelike extremal surfaces, no systematic theory exists for timelike constant (nonzero) mean curvature surfaces. For this reason, we shall devote Section 1 to preliminary materials. The reformulation of the fundamental equations will be carried out in Section 2. As a result we obtain a

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