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Isometric Shift Operators on the Disc Algebra

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Introduction.

The purpose of this note is to study linear isometries on function algebras, especially isometric shift operators on the disc algebra. For a compact Hausdorff space X, we denote by $C(X)$ the Banach space of all complex-valued continuous functions on X. Recently, A. Gutek, D. Hart, J. Jamison and M. Rajagopalan [5] and F. O. Farid and K. Varadarajan [3] have obtained many significant results concerning isometric shift operators on Banach spaces, especially on $C(X)$. Here we investigate linear isometries on function algebras and isometric shift operators on the disc algebra.

In section 1, we give a representation of a codimension 1 linear isometry on a function algebra and in section 2, on the disc algebra A, we establish the form of a codimension 1 linear isometry φ and give equivalent conditions under which φ is a shift operator.

1. Codimension 1 linear isometries on function algebras.

Let E be a Banach space and φ a linear isometry from E into E. Then we call φ a *codimension* 1 *linear isometry* on E if the range of φ has codimension 1. A bounded linear operator φ on E is called a *shift operator* on E if the following conditions are satisfied: (i) φ is injective; (ii) the range of φ has codimension 1; and (iii) $\bigcap_{n=1}^{\infty} \varphi^n(E)=\{0\}$. A linear isometry on E which is a shift operator is an *isometric shift operator* on E.

Let X be a compact Hausdorff space. We say that A is a *function algebra* on X if it is a closed subalgebra of $C(X)$, the Banach algebra of all complex-valued continuous functions on X with the supremum norm, which separates points in X and contains the constants. After now, we consider codimension 1 linear isometries on function algebras and isometric shift operators on the disc algebra.

The following extends a theorem of Gutek, Hart, Jamison and Rajagopalan [5, Theorem 2.1] to the case of the function algebras (cf. [9]).

THEOREM 1.1. *Let A be a function algebra on a compact Hausdorff space X. Suppose*

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