TOKYO J. MATH.
VOL. 21, NO. 1, 1998

Strongly q-Additive Functions and Algebraic Independence

Takeshi TOSHIMITSU

Keio University

(Communicated by Y. Maeda)

1. Introduction.

Let $q\geq 2$ be a fixed integer. A complex-valued function $a(n)$ is said to be *q-additive* or *q-multiplicative* if

$$a(kq^t+r)=a(kq^t)+a(r),\qquad a(0)=0\,, \tag{1}$$

or

$$a(kq^t+r)=a(kq^t)a(r)\,,\qquad a(0)=1 \tag{2}$$

for any integer $k\geq 0$, $t\geq 0$, and $0\leq r<q^t$, respectively. Furthermore, if

$$a(kq)=a(k)\,, \tag{3}$$

$a(n)$ is said to be *strongly q-additive or strongly q-multiplicative*, respectively. We note that the strongly q-additive or q-multiplicative function $a(n)$ is determined completely by the initial values $a(1),\cdots,a(q-1)$. This paper concerns mainly with q-additive functions.

Let $a_1(n),\cdots,a_m(n)$ be m strongly q-additive functions. For each $a_k(n)$, we define a power series $f_k(z)$ by

$$f_k(z):=\sum_{n\geq 0}a_k(n)z^n\in\mathbf{C}[[z]]\qquad (1\leq k\leq m)\,. \tag{4}$$

It follows from (1) and (3) that each $f_k(z)$ converges in $|z|<1$ and satisfies the functional equation

$$f_k(z)=\frac{1-z^q}{1-z}f_k(z^q)+\frac{1}{1-z^q}\sum_{r=0}^{q-1}a_k(r)z^r\qquad (1\leq k\leq m)\,, \tag{5}$$

since

Received July 24, 1996
Revised January 7, 1997