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Kobayashi-Hitchin Correspondence for Perturbed Seiberg-Witten Equations

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1. Introduction.

By theorem of Donaldson-Uhlenbeck-Yau [D2], [UY] there exists a unique irreducible Hermitian-Einstein connection on any stable vector bundle over a compact Kähler manifold. This, together with the result of Kobayashi-Lübke [Ko], [Lü], implies that there is a one-to-one correspondence between the differential geometric and algebro-geometric objects. This correspondence is called Kobayashi-Hitchin correspondence, and can be realized as the relation between the symplectic quotient and stable orbits via the moment map (cf [DK]). The purpose of this paper is to establish a correspondence of Kobayashi-Hitchin type.

Recently Seiberg and Witten introduced new invariants for smooth 4-manifolds. These invariants are defined to be the number of the solutions of the Seiberg-Witten equations. For a closed Kähler surface, there is a correspondence of Kobayashi-Hitchin type between the gauge equivalence classes of irreducible solutions of these equations and a certain type of divisors [W], [FM]. However the moduli spaces of solutions of the unperturbed equations may not be useful to compute the invariants because Kähler metrics are not generic. In [W], Witten introduced a certain perturbation of the Seiberg-Witten equations for Kähler surfaces with $b_2^+ > 1$ to compute the invariants, and he used there the fact that there is also a correspondence of Kobayashi-Hitchin type between the gauge equivalence classes of solutions of the perturbed equations and pairs of divisors. It turns out that the set of these equivalence classes is finite. However there he did not give a proof of this fact. In this paper we shall establish the correspondence for the perturbed equations.

We shall explain the contents of this paper. In section 2, we observe how the unperturbed Seiberg-Witten equations are described for Kähler surfaces. In this situation we find that to each gauge equivalence class of solutions, we can associate an effective divisor. The proof of the bijectivity of this correspondence, which can be considered as

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