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Nonlinear Ergodic Theorems in a Banach Space Satisfying Opials's Condition

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1. Introduction.

Let C be a nonempty closed convex subset of a real Banach space E. Then a mapping $T: C \to C$ is called nonexpansive, if $\|Tx-Ty\| \le \|x-y\|$ for all $x, y \in C$. We denote by $F(T)$ the set of fixed points of T. Let G be a commutative semigroup with identity and let $\mathscr{S}=\{T(s): s \in G\}$ be a family of nonexpansive mappings of C into itself satisfying $T(s+t)=T(s)T(t)$ for all $s, t \in G$, which is called a nonexpansive semigroup on C. Then, $u: G \to C$ is called an almost-orbit of $\mathscr{S}=\{T(s): s \in G\}$ if

$$\lim_s \sup_t \|u(t+s)-T(t)u(s)\|=0 ,$$

where the binary relation $\le$ on G is defined by $a \le b$ if and only if there exists $c \in G$ such that $a+c=b$. The notion of such an almost-orbit was introduced by Takahashi and Park [24]; see Bruck [4] in the case of $G=\{1, 2, 3, \cdots\}$ and Miyadera and Kobayasi [15] in the case of $G=\{t: 0 \le t < \infty\}$.

The first nonlinear ergodic theorem for nonexpansive mappings in a Hilbert space was established by Baillon [1]: Let C be a nonempty closed convex subset of a Hilbert space and let T be a nonexpansive mapping of C into itself. If the set $F(T)$ is nonempty, then for each $x \in C$, the Cesàro means

$$S_n(x)=\frac{1}{n}\sum_{k=0}^{n-1} T^k x$$

converge weakly to some $y \in F(T)$. In Baillon's theorem, putting $y=Px$ for each $x \in C$, P is a nonexpansive retraction of C onto $F(T)$ such that $PT^n=T^nP=P$ for all positive integers n and $Px \in \overline{co}\{T^n x: n=1, 2, \cdots\}$ for each $x \in C$, where $\overline{co}A$ is the closure of the convex hull of A. Takahashi [20, 22] proved the existence of such retractions, "ergodic retractions", for noncommutative semigroups of nonexpansive mappings in a

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