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On Presheaves Associated to Modules

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Introduction.

Let A be a commutative ring with unity. For a subset E of $\operatorname{Spec} A$, we put

$$(1) \qquad S_E = \bigcap_{\mathfrak{p} \in E} (A \backslash \mathfrak{p}) \qquad (S_\varnothing = A) .$$

Then S_E is a saturated multiplicatively closed set.

To an A-module M, we associate a presheaf $\bar{M}$ in the following way. By putting

$$(2) \qquad \bar{M}(U) = S_U^{-1} M$$

for an open subset U of $\operatorname{Spec} A$, we define a presheaf $\bar{M}$ of modules on $\operatorname{Spec} A$. Then

$$(3) \qquad \bar{M}(D(f)) = M_f \qquad \text{for} \quad f \in A ,$$

$$(4) \qquad \bar{M}_\mathfrak{p} = M_\mathfrak{p} \qquad \text{for} \quad \mathfrak{p} \in \operatorname{Spec} A ,$$

where $D(f) = \{\mathfrak{p} \in \operatorname{Spec} A \mid f \notin \mathfrak{p}\}$. Here $\bar{M}$ is not a sheaf in general. But the sheafification of $\bar{M}$ turns out to be the quasi-coherent $\tilde{A}$-module $\tilde{M}$. Then we ask the question: When is the presheaf $\bar{M}$ actually a sheaf?

Noting that $\bar{M}$ is a sheaf if and only if $\bar{M} = \tilde{M}$, we introduce the following three conditions for a ring A:

(S.1) $\bar{M} = \tilde{M}$ for any A-module M.

(S.2) $\bar{\mathfrak{a}} = \tilde{\mathfrak{a}}$ for any ideal $\mathfrak{a}$ of A.

(S.3) $\bar{A} = \tilde{A}$.

Then it is obvious that (S.1) $\Rightarrow$ (S.2) $\Rightarrow$ (S.3).

The main results of this paper are as follows.

THEOREM 1. *Suppose that A is a valuation ring. Then*

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