TOKYO J. MATH.
VOL. 21, NO. 1, 1998

Viscosity Solutions of Hamilton-Jacobi Equations in Smooth Banach Spaces

El Mahjoub EL HADDAD

University of Bordeaux 1

(Communicated by Ma. Kato)

0. Introduction.

In this paper, we investigate existence and uniqueness of viscosity solutions of Hamilton-Jacobi equations in infinite dimensions. We will be concerned with the Cauchy problem

$$\begin{cases} u_t+H(t, x, Du)=0 & \text{on} \quad (0, T]\times X \\ u(0, x)=u_0(x) & \text{on} \quad X. \end{cases} \tag{0.1}$$

Here X is a Banach space, T is a given positive number, $H: [0, T]\times X\times X^*\to \mathbf{R}$ is a uniformly continuous function, $u_0 : X\to\mathbf{R}$ is a given uniformly continuous, $u : [0, T]\times X\to\mathbf{R}$ is the unknown, $u_t=\partial u/\partial t$, Du is the Fréchet derivative of u and X^* is the dual of X. This notion of solution was introduced by M. G. Crandall and P. L. Lions (see [4] and [5]). They studied basic properties of viscosity solutions of first order Hamilton-Jacobi equations in finite dimensions. In [6], M. G. Crandall, L. C. Evans and P. L. Lions reformulated and simplified this work. Afterwards, many other authors were interested in this subject, including, for instance, H. Ishii [12], [13], [15], G. Barles [1], I. Capuzzo Dolcetta [2], I. Capuzzo Dolcetta and H. Ishii [3], and others. Some years later, M. G. Crandall and P. L. Lions studied the Hamilton-Jacobi equations in infinite dimensions; see [7]. They proved uniqueness and existence of viscosity solutions with the geometrical assumptions that the Banach space has the Radon-Nikodym property and a smooth bump function, and some technical assumptions of uniform continuity on the Hamiltonian. The existence of viscosity solutions were established by use of differential games.

Recently R. Deville, G. Godefroy and V. Zizler [10] proved both existence and uniqueness of bounded viscosity solutions for the stationary problem of the form

$$u+H(Du)=f \quad \text{on} \quad X.$$

Received March 27, 1996
Revised October 16, 1996