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On the Existence of a Conjugacy between Weakly Multimodal Maps

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1. Introduction.

In this paper we study the existence of a conjugacy between weakly multimodal maps, which are defined in Definition 1.2, and Hölder continuity of the conjugacy. Throughout this paper let I be a closed interval $[0, 1]$ except §3.1, and n be an integer. We first recall the notion of topological conjugacy.

DEFINITION 1.1. Let $f, g : I \to I$ be two maps. f and g are *topologically conjugate* if there exists a homeomorphism $\varphi : I \to I$ such that

$$\varphi \circ f = g \circ \varphi . \tag{1.1}$$

The map φ is called the *conjugacy* between f and g.

If the map $\varphi : I \to I$ satisfying (1.1) is continuous monotone surjection, f and g are *semi-conjugate*. The map φ is called the *semi-conjugacy*.

The key idea is the following. If we can define the "inverse" g^{-1} in some sense, we have the operator $\mathcal{T}\alpha = g^{-1} \circ \alpha \circ f$ whose fixed point φ, if it exists, should have the equality $g \circ \varphi = \varphi \circ f$. This idea is found in [1] and [2]. We treat with the following class of transformations.

DEFINITION 1.2. The map $f : I \to I$ is *weakly multimodal* if it is continuous and there are points $0 = a_0 \leq b_0 < a_1 \leq b_1 < \cdots < a_{l+1} \leq b_{l+1} = 1$ such that $f|[b_i, a_{i+1}]$ is strictly monotone and $f|[a_i, b_i]$ is a constant function. Assume that the set $\{a_0 = 0, b_0, a_1, b_1, \cdots, a_{l+1}, b_{l+1} = 1\}$ is chosen as small as possible. Let $J_i = [a_i, b_i]$. We say that the J_i's are *flat intervals*.

The following class of maps are known as the l-modal maps, if $a_i = b_i$ for all i. This map is a special case of weakly multimodal maps (cf. [3]).

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