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On an Inverse Problem for Quasilinear Parabolic Equations

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1. Introduction.

Let us consider the initial boundary value problem:

$$u_t - a(x, u)u_{xx} = 0 \quad \text{in} \quad Q_T, \tag{1.1}$$

$$u(x, 0) = 0 \quad \text{for} \quad 0 < x < 1, \tag{1.2}$$

$$u(0, t) = f(t) \quad \text{and} \quad u(1, t) = 0 \quad \text{for} \quad 0 < t < T, \tag{1.3}$$

where $Q_T \equiv \{(x, t) : 0 < x < 1, 0 < t < T\}$.

Assume that the following conditions for a and f are satisfied:
(i) for any finite $M > 0$, $a(x, z) \in C^1([0, 1] \times [-M, M])$,
(ii) for fixed, positive constants ν and μ, $0 < \nu \leq a(x, z) \leq \mu$ on $[0, 1] \times [-M, M]$,
(iii) for fixed, positive constant C, $|\partial_x a(x, z)| + |\partial_z a(x, z)| \leq C$ on $[0, 1] \times [-M, M]$,
(iv) $f \in H^{1+\beta/2}([0, T])$ $(0 < \beta < 1)$ with $f'(t) > 0$ for $0 < t < T$,
(v) $f(0) = 0 = f'(0)$.

From the conditions (i)–(v) applying Theorem 5.2 and Remark 5.1 in [5], we see that there exists a unique solution $u(x, t) \in H^{2+\beta, 1+\beta}([0, 1] \times [0, T])$ to the initial boundary value problem (1.1)–(1.3). So we may define D-N map as follows:

$$\Lambda(a, f) : u(0, t) = f(t) \mapsto u_x(0, t) \quad \text{on} \quad [0, T].$$

We are interested in uniqueness results for $a(x, u)$ of the equation (1.1) from $\Lambda(a, f)$. Isakov [4] proved the uniqueness for $a(x, u)$ in the case that the spatial dimension is greater than or equal to 2 by using the completeness of products of solutions for linear parabolic equations. But in the case that the spatial dimension is one, the completeness of products of solutions has not been proved yet. So we need another method for proving the uniqueness for $a(x, u)$. In [1] it was shown that the coefficient κ of the equation $a(u)u_t = \kappa(a(u)u_x)_x$ was uniquely determined from overspecified boundary data

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