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On Deformations of Einstein-Weyl Structures

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1. Introduction.

Let M be an n-dimensional manifold with a conformal class C. A conformal connection on M is an affine connection D preserving the conformal class C. We also assume D is torsion-free. The triple (M, C, D) is called a *Weyl manifold* or (C, D) is called a *Weyl structure* on M. A Weyl manifold admits an *Einstein-Weyl structure* if the symmetric part of the Ricci curvature of the conformal connection is proportional to a conformal metric which belongs to C. The Einstein-Weyl equations on the metric and affine connection are conformally invariant nonlinear partial differential equations. If (M, g) is an Einstein manifold, then this conformal class C and the Levi-Civita connection defines an Einstein-Weyl structure. So the notion of the Einstein-Weyl manifolds is a generalization of an Einstein metric to conformal structures.

In this paper we consider infinitesimal deformations of an Einstein metric as an Einstein-Weyl structure, and we prove any such deformation comes from conformal Killing vector fields provided certain conditions of curvatures are satisfied.

2. Preliminaries.

Let (M, C, D) be a Weyl manifold. We assume $n=\dim M\geq 3$. This implies the existence of a 1-form ω_g such that $Dg=\omega_g\otimes g$. Let Ric^D denote the Ricci curvature of D. In general, Ricci curvature of conformal connection is not symmetric, so we denote by $\mathrm{Sym}(\mathrm{Ric}^D)$ its symmetric part. The scalar curvature R_g^D of D with respect to $g\in C$ is defined by

$$R_g^D=\mathrm{tr}_g\mathrm{Ric}^D\,. \tag{1}$$

A Weyl manifold (M, C, D) is said to be *Einstein-Weyl manifold* if the symmetric part of the Ricci curvature Ric^D is proportional to the metric g in C. So the

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