TOKYO J. MATH.
VOL. 21, NO. 2, 1998

On the Uniqueness of a Weyl Structure with Prescribed Ricci Curvature

Minyo KATAGIRI

Nara Women's University

(Communicated by T. Nagano)

1. Introduction.

Let M be an n-dimensional manifold with a conformal class C. A conformal connection on M is an affine connection D preserving the conformal class C. We also assume D is torsion-free. The triple (M, C, D) is called a *Weyl manifold* or (C, D) is called a *Weyl structure* on M. In general, the Ricci curvature Ric^D of D is not symmetric, so we denote by $\mathrm{Sym}(\mathrm{Ric}^D)$ its symmetric part.

We consider a problem of a Weyl structure with prescribed Ricci curvature as follows: For a given conformal class C and a (0, 2)-tensor H, can we find a conformal connection D such that $\mathrm{Ric}^D = H$? In this paper, we prove the following result on uniqueness for the problem.

THEOREM 1. *Let M be a closed connected n-manifold, $n \geq 3$, with a conformal class C, and let D and $\bar{D}$ be torsion-free conformal connections of (M, C). If* $\mathrm{Sym}(\mathrm{Ric}^D) = \mathrm{Sym}(\mathrm{Ric}^{\bar{D}})$, *then $D = \bar{D}$.*

The result shows for a conformal connection, the symmetric part of the Ricci curvature determines the full Ricci curvature. The following corollary is due to [7].

COROLLARY 2. *Let (M, C, D) be a closed connected Weyl n-manifold, $n \geq 3$. If* $\mathrm{Sym}(\mathrm{Ric}^D) = \mathrm{Ric}_g$ *for some Riemannian metric $g \in C$, then D is the Levi-Civita connection of g, and such a g is unique in C up to a constant multiple.*

2. Preliminaries.

Let (M, C, D) be a Weyl manifold. We assume $n = \dim M \geq 3$. Then there is a unique 1-form ω_g such that $Dg = \omega_g \otimes g$.

We denote by Ric^D the Ricci curvature of D, and by $\mathrm{Sym}(\mathrm{Ric}^D)$ the symmetric part of the Ricci curvature. The scalar curvature R_g^D of D with respect to $g \in C$ is defined

Received April 8, 1997