TOKYO J. MATH.
VOL. 21, NO. 2, 1998

Estimates for the Operators $V^{\alpha}(-\Delta+V)^{-\beta}$ and $V^{\alpha}\nabla(-\Delta+V)^{-\beta}$ with Certain Non-negative Potentials V

Satoko SUGANO

Gakushuin University

(Communicated by K. Akao)

1. Introduction and main results.

Let $V\in L^1_{loc}(\mathbf{R}^n)$, $n\geq 3$, be a non-negative potential and consider the Schrödinger operator $L=-\Delta+V$. If V belongs to the reverse Hölder class B_q, the L^p boundedness of the operators VL^{-1}, $V^{1/2}L^{-1/2}$, $V^{1/2}\nabla L^{-1}$, and $\nabla L^{-1/2}$ were proved by Shen ([Sh]). For operators of the type VL^{-1} and $V^{1/2}\nabla L^{-1}$, these results were generalized as follows ([KS]). We replace Δ by the second order uniformly elliptic operator L_0 and let $L=L_0+V$. Suppose V satisfy the same condition as above. Then, the operators VL^{-1} and $V^{1/2}\nabla L^{-1}$ are bounded on weighted L^p spaces and Morrey spaces.

The purpose of this paper is to extend Shen's results to another direction. More precisely, we shall investigate the L^p-L^q boundedness of the operators

$$T_1=V^{\alpha}(-\Delta+V)^{-\beta}\,,\qquad 0\leq\alpha\leq\beta\leq 1\,,$$
$$T_2=V^{\alpha}\nabla(-\Delta+V)^{-\beta}\,,\qquad 0\leq\alpha\leq 1/2\leq\beta\leq 1\,,\quad \beta-\alpha\geq 1/2$$

on $\mathbf{R}^n$, $n\geq 3$. We obtain weighted estimates for T_1 and T_2 and their boundedness on Morrey spaces.

Shen established the estimate of the fundamental solution of the Schrödinger operator by using the auxiliary function $m(x, V)$ which was introduced by himself. One of his idea is to combine the estimates of the fundamental solution with the technique of decomposing $\mathbf{R}^n$ into spherical shells $\{x\,|\,2^{j-1}r<|x|\leq 2^j r\}$, $r=\{m(x, V)\}^{-1}$, for estimating various integral operator (see [Sh, Theorem 4.13, Theorem 5.10]). We shall prove our theorems by similar methods.

As we mentioned above, for the special values of α, β, the estimate for T_1 and T_2 were proved in [Sh] and [KS]. For the operator T_2, our theorem does not cover the case $(\alpha, \beta)=(0, 1/2)$. To prove this case Shen's advanced method is needed (see [Sh, Theorem 0.5] and its proof).

In their paper ([Sh], [KS]), the authors obtained pointwise estimates for the

Received March 26, 1997