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Initial Value Problems for the Heat Convection Equations in Exterior Domains

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1. Introduction.

We consider the initial value problem (IVP) of the heat convection equation (HCE) of Boussinesq type in an exterior domain $\Omega=K^c\subset\mathbf{R}^m$ ($m=2$ or 3), where K is a compact set with a smooth boundary $\Gamma=\partial K\in C^2$. We denote $\hat{\Omega}=\Omega\times(0,T)$. Then the problem (IVP) for (HCE) is as follows:

$$\begin{cases} u_t+(u\cdot\nabla)u=-(\nabla p)/\rho+\{1-\alpha(\theta-\Theta_0)\}g+\nu\Delta u & \text{in } \hat{\Omega}\,, \\ \operatorname{div} u=0 & \text{in } \hat{\Omega}\,, \\ \theta_t+(u\cdot\nabla)\theta=\kappa\Delta\theta & \text{in } \hat{\Omega}\,, \end{cases} \tag{1}$$

$$u|_\Gamma=0\,,\quad \theta|_\Gamma=\Theta_0>0\,,\quad \lim_{|x|\to\infty}u(x,t)=0\,,\quad \lim_{|x|\to\infty}\theta(x,t)=0 \text{ for } t\in(0,T)\,, \tag{2}$$

$$u|_{t=0}=a\,,\qquad \theta|_{t=0}=h\,. \tag{3}$$

Here, $u=u(x,t)$ is the velocity vector, $p=p(x,t)$ is the pressure and $\theta=\theta(x,t)$ is the temperature; ν, κ, α, ρ and $g=g(x)$ are the kinematic viscosity, the thermal conductivity, the coefficient of volume expansion, the density at $\theta=\Theta_0$ and the gravitational vector, respectively.

Hishida [2] and Hishida-Yamada [3] studied the exterior problem for (HCE) and proved the global existence of the strong solution of (IVP) when K is a ball, while the second author of our present paper has recently shown in [6] (which is her Master thesis) the existence of a weak solution of (IVP) for (HCE) in the case that K is a compact set with a smooth boundary of class C^2. In Ōeda-Matsuda [10], we announced the existence result ($m=2$ or 3) together with the uniqueness of a weak solution for the 2-dimensional problem. In the present paper, we will give details of proofs of the results announced in [10], and furthermore, show the uniqueness theorem of a weak solution for the 3-dimensional problem. As the equations (1) tell us, (HCE) is the system which

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