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Weyl Groups of the Extended Affine Root System $A_1^{(1,1)}$ and the Extended Affine $\mathfrak{sl}(2)$

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1. Introduction.

The notion of the extended affine root systems was introduced by K. Saito ([1]). These root systems relate to the simple elliptic singularity, and by definition, they are extensions of the affine root systems by one dimensional radical. The Weyl groups of the extended affine root systems and their hyperbolic extension groups have been also studied in [1], [2], from a geometric point of view. In this paper, in the case of $A_1^{(1,1)}$, from the view point of representation theory, we study the Weyl group of $A_1^{(1,1)}$ and its hyperbolic extension group, which we denote by $W(A_1^{(1,1)})$ and $\tilde{W}(A_1^{(1,1)})$, respectively. From their constructions, we find that $\tilde{W}(A_1^{(1,1)})$ ([1]) is isomorphic to the Weyl group $W(\hat{\mathfrak{sl}}_2)$ of the extended affine Lie algebra $\hat{\mathfrak{sl}}_2$ (the extended affine $\mathfrak{sl}(2)$). At first, we fix the generators of $W(A_1^{(1,1)})$ and decide their relations by considering $W(A_1^{(1,1)})$ as an extension of the finite Weyl group $W(A_1)$ by translations of two directions, and further using this result, we show that $W(A_1^{(1,1)})$ contains the infinite dihedral group D_∞ as a subgroup and is an extension of the dihedral group D_2. Further we present that $W(A_1^{(1,1)})$ is non-amenable. The extended affine Lie algebra $\hat{\mathfrak{sl}}_2$ is defined as follows. In quantum field theory, the gauge group (the current group) is defined to be the set of smooth functions from the compact manifold M onto the semi-simple compact Lie group G with a pointwise product. When $M=T^\nu=S^1\times\cdots\times S^1$ i.e. ν-dimensional torus, the corresponding Lie algebra is the gauge algebra $P(T^\nu, \mathfrak{g})$, where $\mathfrak{g}$ is the Lie algebra of G and $P(T^\nu, \mathfrak{g})$ means the set of functions from T^ν into $\mathfrak{g}$ with finite fourier series. The central extensions of $P(T^\nu, \mathfrak{g})$ are infinite dimensional Lie algebras, and called quasi-simple Lie algebras ([16]). Especially, when $\nu=1$, $P(S^1, \mathfrak{g})$ is usually written as $\mathbf{C}[t, t^{-1}]\otimes\mathfrak{g}$, where $\mathbf{C}[t, t^{-1}]$ is the ring of Laurent polynomials in t, and its central extension is called affine Lie algebra (Kac-Moody algebra). Then the corresponding Lie group is called loop group ([20]). Further let $M=T^2=S^1\times S^1$, then the Lie algebra

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