Another Type of Instanton Bundles on $Gr_2(\mathbf{C}^{n+2})$

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1. Introduction.

The purpose of the present paper is to show the existence of an anti-self-dual connection on 2-plane complex Grassmannian $Gr_2(\mathbf{C}^{n+2})$, to classify instantons and to describe the moduli space. The reason why we use the terminology "instanton" is that our anti-self-dual connections are nothing but 1-instantons in the case $n=1$ $(\mathbf{C}P^2)$. However we also have proved that there exists another generalization of instantons on $\mathbf{C}P^2$ to $Gr_2(\mathbf{C}^{n+2})$ [N-N2]. The structure group reflects the main difference between them. In the present paper, $SU(r)$-bundles are taken into account, while in [N-N2], $Sp(r)$-bundles are considered. By the isomorphism $SU(2) \cong Sp(1)$, our two series of generalizations coincide with instantons on $\mathbf{C}P^2$ in the case $n=1$. On $\mathbf{H}P^n$, which is another typical example of quaternion-Kähler manifolds, there exists a generalization of instantons on 4-dimensional sphere $S^4 \cong \mathbf{H}P^1$. This instanton bundle also has $Sp(r)$ as a structure group and so, odd Chern classes of this bundle vanish. Since the cohomology groups $H^{4i}(\mathbf{H}P^n, \mathbf{Z}) \cong \mathbf{Z}$ for $i = 0, 1, \cdots, n$ and the others vanish, odd Chern classes of an arbitrary bundle on $\mathbf{H}P^n$ necessarily vanish. On the contrary, our examples have the non-vanishing third Chern classes. In higher dimensional case, these are the first examples such that higher degree odd Chern classes do not vanish.

As for the existence of anti-self-dual connections, Mamone-Capria and Salamon first give the above examples of instanton bundles on $\mathbf{H}P^n$ and prove that a well-known Horrocks bundle on $\mathbf{C}P^5$ can be obtained as the pull-back of an anti-self-dual bundle on $\mathbf{H}P^2$ [M-S]. Applying the monad given by Donaldson [D] to higher dimensional case, $Sp(r)$-instanton bundles on $Gr_2(\mathbf{C}^{n+2})$ are exhibited in [N-N2]. In both cases, the typical examples of 1-instantons are homogeneous bundles with canonical connections. The author determines all irreducible homogeneous bundles with anti-self-dual canonical connections over compact quaternion symmetric spaces and give a deformation of canonical connections [Na-3]. Adapting this point of view, we will deform the canonical connection on a direct sum of a line bundle and a homogeneous bundle on $Gr_2(\mathbf{C}^{n+2})$.

To classify anti-self-dual bundles, we make use of the theory of monads on the