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On Log Canonical Thresholds of Surfaces in $\mathbf{C}^3$

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1. Introduction.

There are some invariants to measure how singular a hypersurface is. The log canonical threshold is useful for classifying non log canonical pairs. In this paper, we will study hypersurface singularities in $\mathbf{C}^n$. The sigularities are not necessarily isolated.

Let f be a holomorphic function near $0 \in \mathbf{C}^n$, and let $D = div(f)$. Then the log canonical threshold of f at 0 is defined by

$$c_0(\mathbf{C}^n, f) := sup\{c : (\mathbf{C}^n, cD) \text{ is log canonical near } 0\}.$$

We frequently write $c_0(f)$ instead of $c_0(\mathbf{C}^n, f)$, if there is no confusion. If $f \neq 0$, then we see that $0 < c_0(f) \leq 1$ and $c_0(f) \in \mathbf{Q}$. The log canonical threshold $c_0(f)$ is an interesting number, because it has some equivalent definitions (See Chapter 8, 9, 10 in [4] for a detailed explanation):

1. $c_0(f) = sup\{c : |f|^{-c} \text{ is locally } L^2 \text{ near } 0\}$
2. $c_0(f) = -$(the largest root of the Bernstein-Sato polynomial of f).

Shokurov proposed the following conjecture in [8].

CONJECTURE. *Let $\mathscr{T}_n = \{c_0(\mathbf{C}^n, f) : f \neq 0$ is holomorphic near $0 \in \mathbf{C}^n\}$. For every $n \in \mathbf{N}$, the set $\mathscr{T}_n$ satisfies the ascending chain condition.*

Shokurov proved it for the case $n = 2$ in the paper. And this was proved for the case $n = 3$ by Alexeev [2].

It is an interesting problem to describe $\mathscr{T}_n$ explicitly. By Shokurov's method, we can find $\mathscr{T}_2$ (Lemma 3.1), but Alexeev's method is ineffective and gives us little information about $\mathscr{T}_3$. So the case $n \geq 3$ is unknown. We do not even know the accumulation points of $\mathscr{T}_3$.

The aim of this paper is to show the following theorem.

THEOREM. *Let f be a nonzero holomorphic function near $0 \in \mathbf{C}^3$. Then all the list of $\{c_0(\mathbf{C}^3, f) \geq 5/6\}$ are as follows:*

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