

Stability of Singular Leaves of Compact Hausdorff Foliations with Tori as Generic Leaves

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(Communicated by K. Akao)

Introduction.

A codimension q C^r foliation \mathcal{F} of a closed manifold M is said to be C^r -stable if there exists a neighbourhood V of \mathcal{F} in the set of codimension q C^r foliations, which carries a natural weak C^r -topology (cf. Hirsch [7], Epstein [2]), such that every foliation in V has a compact leaf. Kazuhiko Fukui has studied the stability of foliations of closed manifolds by Klein bottles ([4], [5]). In this paper, we study the stability of Hausdorff C^r ($1 \leq r \leq \infty$) foliations of closed manifolds of dimension n ($n=4, 5$) with tori as generic leaves, where a foliation \mathcal{F} is said to be Hausdorff if the leaf space M/\mathcal{F} is Hausdorff. Epstein showed if a foliation \mathcal{F} is Hausdorff, there is a generic leaf L_0 with the property that there is an open dense saturated subset of M where all leaves have trivial holonomy and are diffeomorphic to L_0 (cf. Epstein [2] and also §1). A leaf of \mathcal{F} is said to be singular if it has non-trivial holonomy group. We shall classify the types of the singular leaves (Theorems 4, 5) and discuss their local stabilities (Theorems 9, 13 in the case of codimension 2 foliations and Theorems 15, 18 for codimension 3).

The author would like to thank Prof. K. Fukui for fruitful discussions through his papers and also to thank Prof. Takashi Inaba for his helpful advice.

1. Local behaviour of foliations.

Let \mathcal{F} be a codimension q compact Hausdorff C^r ($1 \leq r \leq \infty$) foliation of a closed manifold M . On the local behaviour of \mathcal{F} , there are results of Epstein [2] and Edwards-Millett-Sullivan [1] and it can be described after Fukui ([4], [5]) as follows:

PROPOSITION 1 (Epstein [2] Thm. 4.3). *There is a generic leaf L_0 with the property that there is an open dense saturated subset of M where all leaves have trivial holonomy and are diffeomorphic to L_0 . Given a leaf L , we can describe a neighbourhood $U(L)$ of L , together with the foliation on the neighbourhood as follows. There is a finite subgroup*